Wayne
Township Public Schools

Mathematics

Grades 9 - 10

Geometry

Dr. Mark Toback, Superintendent

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This curriculum may be modified through varying techniques, strategies, and materials as per an individual student's Individualized Educational Plan (IEP)

Approved by the Wayne Township Board of Education at the regular meeting held on November 15, 2018.

Wayne Public Schools:
Mathematics
Curriculum

Course Title: **GEOMETRY**	**Grade Level: 9/10**

UNIT 1: Basics of Geometry: 8 weeks

UNIT 2: Transformation and Congruence: 5-6 weeks

UNIT 3: Similarity, Right Triangles, and Trigonometry: 7 weeks

UNIT 4: Circles: 3-4 weeks

UNIT 5: Geometric Measurement and Dimension: 3-4 weeks

NOTE: All pacing is APPROXIMATE and allows for review and assessment days

Date Updated: **Winter 2017**

Updated by: Jason Wilke, Zach Smith, MarySue Rossinow, Andrew Poalillo

NEW JERSEY STUDENT LEARNING STANDARDS FOR MATHEMATICS

Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning , strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

Mathematically proficient students:

- explain to themselves the meaning of a problem and looking for entry points to its solution.
- analyze givens, constraints, relationships, and goals.
- make conjectures about the form and meaning of the solution attempt.
- consider analogous problems, and try special cases and simpler forms of the original problem.
- monitor and evaluate their progress and change course if necessary.
- transform algebraic expressions or change the viewing window on their graphing calculator to get information.
- explain correspondences between equations, verbal descriptions, tables, and graphs.
- draw diagrams of important features and relationships, graph data, and search for regularity or trends.
- use concrete objects or pictures to help conceptualize and solve a problem.
- check their answers to problems using a different method.
- ask themselves, "Does this make sense?"
- understand the approaches of others to solving complex problems.

2 Reason abstractly and quantitatively.

Mathematically proficient students:

- make sense of quantities and their relationships in problem situations.
  - ➢ *decontextualize* (abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents and
  - ➢ *contextualize* (pause as needed during the manipulation process in order to probe into the referents for the symbols involved).
- use quantitative reasoning that entails creating a coherent representation of quantities, not just how to compute them
- know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students:

- understand and use stated assumptions, definitions, and previously established results in constructing arguments.
- make conjectures and build a logical progression of statements to explore the truth of their conjectures.
- analyze situations by breaking them into cases
- recognize and use counterexamples.
- justify their conclusions, communicate them to others, and respond to the arguments of others.
- reason inductively about data, making plausible arguments that take into account the context
- compare the effectiveness of plausible arguments
- distinguish correct logic or reasoning from that which is flawed
  - ➢ elementary students construct arguments using objects, drawings, diagrams, and actions..
  - ➢ later students learn to determine domains to which an argument applies.
- listen or read the arguments of others, decide whether they make sense, and ask useful questions

4 Model with mathematics.

Mathematically proficient students:

- apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
  - ➢ In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.
  - ➢ By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another.
- simplify a complicated situation, realizing that these may need revision later.
- identify important quantities in a practical situation
- map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas.
- analyze those relationships mathematically to draw conclusions.
- interpret their mathematical results in the context of the situation.
- reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students

- consider available tools when solving a mathematical problem.
- are familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools
- detect possible errors by using estimations and other mathematical knowledge.
- know that technology can enable them to visualize the results of varying assumptions, and explore consequences.
- identify relevant mathematical resources and use them to pose or solve problems.
- use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students:

- try to communicate precisely to others.
- use clear definitions in discussion with others and in their own reasoning.
- state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.
- specify units of measure and label axes to clarify the correspondence with quantities in a problem.
- calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the context.
  - ➢ In the elementary grades, students give carefully formulated explanations to each other.
  - ➢ In high school, students have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students:

- look closely to discern a pattern or structure.
  - ➢ Young students might notice that three and seven more is the same amount as seven and three more.
  - ➢ Later, students will see 7 x 8 equals the well-remembered 7 x 5 + 7 x 3, in preparation for the distributive property.
  - ➢ In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 x 7 and the 9 as 2 + 7.
- step back for an overview and can shift perspective.
- see complicated things, such as some algebraic expressions, as single objects or composed of several objects.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students:

- notice if calculations are repeated
- look both for general methods and for shortcuts.
- maintain oversight of the process, while attending to the details.
- continually evaluate the reasonableness of intermediate results.

Wayne School District
Curriculum

Content Area/ Grade Level/ Course:	**Mathematics** **9/10** **Geometry**
Unit Plan Title:	**Unit 1: Basics of Geometry**
Time Frame	**9 Weeks**

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Number and Operations in Base 10

G-CO – Geometry: Congruence, Proof, and Constructions

Unit Summary

Students will be able to identify the basic foundational concepts in geometry such as points, lines, and planes. They should be able to use, construct, and solve for missing information in angles, segments, transversals, and basic polygons. They should also be able to prove theorems using a variety of formats, and solve problems about quadrilaterals.

Standard Number(s)

G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G-CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G-GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$

G-GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G.GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

8.1.12.CS.4 - Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1 - Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5 - Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.DA.6 - Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.

9.4.12.CI.1 - Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).

9.4.12.CT.2 - Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)

9.4.12.TL.2 - Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.IML.3 - Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

RST.9-10.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

RST.9-10.5. Analyze the relationships among concepts in a text, including relationships among key terms (e.g.,force,, friction, reaction force, energy).

RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

WHST.9-10.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

WHST.9-10.5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.

WHST.9-10.6. Use technology , including the Internet, to produce, share, and update writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

Essential Question(s)

- How can you make a conjecture and prove that it is true?
- How do you construct a logical argument?
- How do you write a bi-conditional statement?
- How do you use inductive and deductive reasoning?
- How do you write a geometric proof?
- How can you determine if two lines are parallel? Perpendicular?
- How can you classify a quadrilateral given its coordinates?
- How can you determine a point is the midpoint of a segment?

Enduring Understandings

- Patterns and relationships can be observed in number sequences and geometric figures.
- Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa.

In this unit plan, the following 21st Century themes and skills are addressed.

	Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**
x	**Global Awareness**	E	**Creativity and Innovation**
	Environmental Literacy	T-A	**Critical Thinking and Problem Solving**
	Health Literacy	T-A	**Communication**
	Civic Literacy	E	**Collaboration**

x	**Financial, Economic, Business, and Entrepreneurial Literacy**

Student Learning Targets/Objectives (Students will know/Students will understand)

- Understand how parallel lines and transversals interact together
- Explore the differences and similarities between different quadrilaterals
- Define, describe, and represent relationships between basic geometric terminology
- Prove theorems about lines, angles, triangles, and parallelograms.
- Make formal geometric constructions with a variety of tools and methods.
- Students will understand how to use the distance formula to find lengths and determine relationships between segments.
- Students will understand to use the midpoint formula to find the midpoint or an endpoint of a segment.
- Students will understand the slope of a line and how to find the slope of a line.
- Students will understand how to determine if lines are parallel and perpendicular and find slopes of parallel and perpendicular lines.
- Students will understand how to prove lines perpendicular.
- Students will understand how to complete coordinate proofs.
- Students will understand how to find a coordinate that partitions a segment with a given ratio.

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Quizzes: A quiz per standard

- **Identify and work with missing information in lines, rays, segments, and angles (G-CO.A.1)**
- **Prove Geometric theorems. (G-CO.C.9 – G-CO.C.11)**
- **Make geometric constructions. (G-CO.D.12)**

Unit Tests:

- **A test covering all topics of geometric basics, transversals, and polygons respectively.**

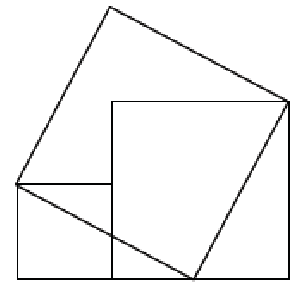
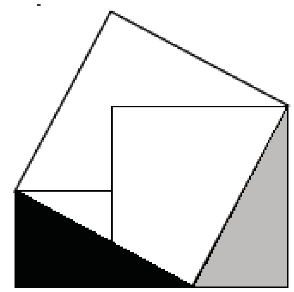
***Unit Project*:**

- **For example, a project on tessellations in which students complete various transformations and constructions of geometric figures.**

***Quarterly*:**

- **Benchmark tests.**

Teaching and Learning Activities	
	****Interactive applets for G-CO** http://www.regentsprep.org/regents/math/geometry/GT2/Activity.htm ** Excellent intro activity. http://nlvm.usu.edu/en/nav/topic_t_3.html (a variety of geometry topics) http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php http://www.mathopenref.com/parallelogram.html - Parallelogram information (others, too) http://www.mathopenref.com/transversal.html - Transversals http://www.mathopenref.com/angle.html - Angle information http://www.mathwarehouse.com/geometry/quadrilaterals/ - Interactive Quadrilateral Family tree ****Interactive Applets and activities for G-GPE** (1) Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram. (2) Recognizing a square (The Mathematics Assessment Program (MAP)) http://map.mathshell.org/materials/tasks.php?taskid=270&subpage=apprentice (3) Classify Polygons using coordinate geometry: http://psdsecondarymath1.pbworks.com/w/file/fetch/48303916/SM1%2010-3.doc (4) (Formative assessment) Finding equations of parallel and perpendicular lines Finding Equations of Parallel and Perpendicular Lines (5) Explore learning on distance formula: http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=183 (6) Lesson plan on quadrilaterals: http://alex.state.al.us/lesson_view.php?&print=friendly&id=23995 (7) (Polygon Area Calculator) (Coordinate Geometry) http://www.mathopenref.com/coordpolygonareacalc.html (8) Coordinate Geometry (lessons and assessments from New York State Regents resources) ■ Slopes and Equations of Lines ■ Midpoint of a Line Segment ■ Distance Formula ■ Direct Analytic Proofs (Coordinate Geometry Proofs) ■ Circles ■ Multiple Choice Practice - Coordinate Geometry *(interactive & hardcopy)*

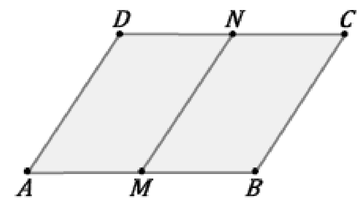
	(10) Multiple choice problems from http://www.glencoe.com/sec/math/geometry/geo/geo_05/chapter_test/index.php/nj/2006
Exemplars	A company has designed a new logo using overlapping squares.  1. How many squares do you see in the logo? ______________________ Describe where you see the squares.  2. The logo designer colored two triangles in the logo How are the two triangles related? Justify your answer. 3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

G.CO.11 – Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Resources: Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.

Example:

- Suppose that $ABCD$ is a parallelogram, and that M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively. Prove that $MN = AD$, and that the line $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{AD}$.

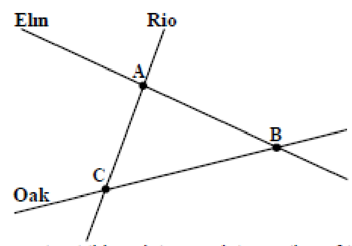


G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Resources: Students may use geometric software to make geometric constructions.

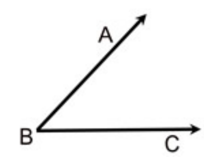
Examples:

- Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.
- Construct the circumventer of a given triangle.
- You have been asked to place a warehouse so that it is an equal distance from the three roads indicated on the following map. Find this location and show your work.



- Show how to fold your paper to physically construct this point as an intersection of two creases.
- Explain why the above construction works and, in particular, why you only needed to make two creases.

Given: $\angle ABC$
Construct: an angle congruent to $\angle ABC$.
(*make a copy of the angle*)



Differentiation Strategies and Activities

Use interactive Applets (see ** above) to appeal to the visual learner.

Activities

1. Introducing Vertical angles:

- Use projector to utilize the website: http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php
- Manipulate the vertical angles to demonstrate the congruent angles regardless of the shape the X makes.
- Have students hypothesize the measure of the other two angles.

2. Introducing 2 Parallel lines cut by a transversal (Labeling/Understanding locations)

- Make up 2 sets of cards- 3x5 index cards cut in half work well
- Label one set in red lettering with pairs labeled “Corresponding 1”, “Corresponding 2”, etc
- Label on set in blue lettering with pairs labeled “Alt. Int. 1”, “Alt. Int. 2”, “Alt. Ext. 1” &”Alt. Ext. 2”
- Use electrical tape to make a set of the 3 lines on the floor for each group of students.
- Have students “label” their diagram with the red set first.
- Walk around to check correctness.
- Do the same with the blue set.
- Alternate students and colors to ensure each student practices with each color set.

3. Introducing Triangle Sum Theorem

- Give each group 3 different types of triangles made from construction paper.
- Have students label the 3 angles as A, B, and C.
- Have students rip the triangles into 3 pieces-each piece has to contain an entire angle.
- Have students tape the 3 angles against each other by the “points” of the angles.
- Ask students:
  - What is formed by the 3 angles when taped in this manner? (it’s a line)
  - What do we know about the measure of a straight angle or a line?

Student will be able to determine that the 3 angles of any type of triangle creates a line which is why the interior sum equals 180˚.

4. **Introducing Midpoint formula**

- Each group of students is given a double sided 1 cm grid paper and directions to draw vertical and horizontal lines of given lengths (2 “even” and 2 “odd”).
- Have students estimate what they believe is the midpoint and label it as “M”.
- Have students:
  - ➢ Label page with an x axis and y axis to make Quadrant 1
  - ➢ Identify the coordinates of the endpoints of the segments.
  - ➢ Label points with x_1, y_1 and x_2, y_2
  - ➢ Substitute values into the midpoint formula and derive the answer.
  - ➢ Ask students to draw conclusions about the pairs of answers.
- Have students flip to the other side and given them 3 pairs of coordinates to plot and then connect to form 3 diagonal lines.
- Ask the students to ask what difficulties they would have with finding the midpoint of the segments as compared to the first set.
- Have students measure the lengths with the ruler & estimate midpoint, then substitute the coordinates into the distance formula and derive the exact answer.

5. **Introducing Slope**

- Each group of students is given Geoboards and rubber bands. Have students make positive ‘lines’ with the rubber bands and determine the slope by counting.
- Have students make determinations as to what happens to the slope number as the lines become more or less steep.
- Have students make negative ‘lines’ by the same method.
- Ask them to try to make a line parallel and/or perpendicular to the line. What do they notice about the slopes?
- Have students try to make 2 lines with one positive and one negative of the same slope.

- Give students graph paper with an x axis and y axis and the 4 quadrants.
- Have students replicate a slope line from the Geoboard onto the graph paper and find 2 coordinates on the line.
- Give students the slope formula and have them substitute values in to determine if they have the same answer for the slope both ways.

Differentiation Strategies:

- All students should be evaluated on the basic mathematical requirements of each topic.
- Most students should be evaluated on the basic algebraic requirements of each topic.
- Some students should be evaluated on the complex (multi-step) algebraic requirements of each topic.

As an example using 2 lines cut by a transversal:

- All students should be able to determine the other 7 angle measures when given 1 angle measure.
- Most students should be able to solve for x, given a variable expression for 1 angle measure and the measure of another angle.
- Some students should be able to solve for x and give the angle measure when given variable expressions for 2 different angles.

Differentiation Strategies for Special Education Students
Differentiation Strategies for Gifted and Talented Students
Differentiation Strategies for ELL Students
Differentiation Strategies for At Risk Students

Honors:

The students enrolled in this level will be assigned questions of greater complexity

Standard	Fundamental	Average	Advanced
G-CO.A.1 G-CO.A.2	Naming points, lines, and planes.	Describe the difference between congruence and equal	Construct angle bisectors and perpendicular bisectors
G-CO.D.12	Identify solids from a net.	Draw a solid from a net	Identify isometric drawings and orthographic drawings
G-CO.D.13	Use the isosceles triangle theorem to find missing angle measures.	Use Algebra to find values of all triangles especially isosceles triangles	Write a proof for the isosceles triangles theorem converse

Resources

- **Teacher Edition Textbook**
- **Online Resources through McDougal Littell Classzone**

- **Real world Applications of Geometry**
- **KahnAcademy.com**
- **KUTA**
- **Edhelper.com**
- **Explore Learning**
- **Miras**
- **Protractors**
- **Geometer's Sketchpad**

Online resources listed above in "Activities"

Wayne School District
Curriculum

Content Area/ Grade Level/ Course:	**Mathematics** **9/10** **Geometry**
Unit Plan Title:	**Unit 2: Transformations and Congruence**
Time Frame	**4 Weeks**

Anchor Standards/Domain*   *i.e: ELA: reading, writing i.e.: Math: Number and Operations in Base 10

G- CO – Geometry: Congruence, Proof, and Constructions

Unit Summary

Students will be able to establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They should be able to use triangle congruence as a familiar foundation for the development of formal proof. Students should prove theorems using a variety of formats, and solve problems about triangles, quadrilaterals and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Standard Number(s)

G-CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs.
Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G-CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G-CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G-CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G-CO.C.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

G-CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G-CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

G-CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

G-CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

8.1.12.CS.4 - Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1 - Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5 - Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.DA.6 - Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.

9.4.12.CI.1 - Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).

9.4.12.CT.2 - Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)

9.4.12.TL.2 - Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.IML.3 - Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

RST.9-10.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

RST.9-10.5. Analyze the relationships among concepts in a text, including relationships among key terms (e.g.,force,, friction, reaction force, energy).

RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

WHST.9-10.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

WHST.9-10.5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.

WHST.9-10.6. Use technology , including the Internet, to produce, share, and update writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

Essential Question(s)

- How can you make a conjecture and prove that it is true?
- How do you identify corresponding parts of congruent triangles?
- What are congruent segment/angles?
- How do you construct a logical argument?
- How do you write a bi-conditional statement?
- How do you use inductive and deductive reasoning?
- How do you write a geometric proof?
- How are corresponding angles, and alternate interior angles related?
- What are congruent figures?
- How can you use side lengths and angle measurements to prove triangles are congruent?
- How can you use congruent triangles to prove angles or sides are congruent?
- What transformations create an image congruent to the original figure?

Enduring Understandings

- Patterns and relationships can be observed in number sequences and geometric figures.

In this unit plan, the following 21st Century themes and skills are addressed.			
Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
x	**Global Awareness**	x	**Creativity and Innovation**
	Environmental Literacy	x	**Critical Thinking and Problem Solving**
	Health Literacy	x	**Communication**
	Civic Literacy	x	**Collaboration**
x	**Financial, Economic, Business, and Entrepreneurial Literacy**		

Student Learning Targets/Objectives (Students will know/Students will understand)

- **Experiment with transformations in the plane. (G.CO.1 – G.CO.5)**
- **Understand congruence in terms of rigid motions. (G.CO.6 – G.CO.8)**
- **Define, describe, and represent congruency of figures in terms of rigid motions.**
- **Explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motion.**
- **Prove theorems about lines, angles, triangles, and parallelograms**.

Make formal geometric constructions with a variety of tools and methods.

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Quizzes: A quiz per standard

- **Experiment with transformations in the plane (G-CO.A.1 – G-CO.A.5)**
- **Understand congruence in terms of rigid motions. (G-CO.B.6 – G-CO.B.8)**
- **Prove Geometric theorems. (G-CO.C.9 – G-CO.C.11)**
- **Make geometric constructions. (G-CO.D.12 – G-CO.D.13)**

Unit Test:

- **A test covering all topics of congruence, proof, and construction.**

Unit Project:

- **For example, a project on tessellations in which students complete various transformations and constructions of geometric figures.**

Quarterly:

- **Benchmark tests.**

	Teaching and Learning Activities
	Use Miras to perform constructions of reflecting points, polygons, parallel and perpendicular lines, angle and segment bisectors. http://www.mathchamber.com/PDFs/math8/KHM/KHM%201.1A%20Using%20the%20MIRA.pdf http://www.shawnee.edu/acad/ms/ENABLdocs/Summer08pdfs/MIRA%20Lesson%20Plan.pdf http://wveis.k12.wv.us/teach21/public/Uplans/LPview.cfm?page=1&tsele1=2&tsele2=117&upidU=1528&UPid=1530 http://wveis.k12.wv.us/teach21/public/Uplans/UPview.cfm?action=V1&tsele1=2&tsele2=117&tsele3i=1528 Experience visual learning using Gizmos (www.ExploreLearning.com)* on Dilations, Reflections, Rotations, Similar Figures/Polygons, and Congruence in all polygons. Demonstrations on Congruence with Transformations using The Geometer Sketch Pad. *Reflections http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=194 *Rotations, reflections, translation http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=269 *Similar Figures http://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=296 *Congruence in right triangles http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=179 *Proving triangles are congruent http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=192 *Similar figures Activity A http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=271 *Similar Polygons http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=195 *Similarity in Right Triangles http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=196 *Rock Art (Transformations) http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=1031 **<u>**Interactive applets:**</u>** http://www.mathsisfun.com/flash.php?path=%2Fgeometry/images/translation.swf&w=670.5&h=571.5&col=%23FFFFFF&title=Geometry+Translation (gives general idea of the translation) http://www.regentsprep.org/regents/math/geometry/GT2/Activity.htm ** Excellent intro activity. http://www.shodor.org/interactivate/activities/Transmographer/

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http://www.misterteacher.com/alphabetgeometry/reflection.html (scroll down to "Practice" then click on the grid on the bottom)

http://nlvm.usu.edu/en/nav/topic_t_3.html (a variety of geometry topics)

http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php

http://www.analyzemath.com/Geometry/MediansTriangle/MediansTriangle.html

http://www.mathwarehouse.com/geometry/quadrilaterals/parallelograms/interactive-parallelogram.php

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Exemplars

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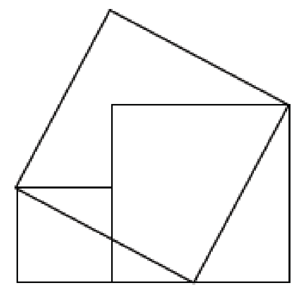
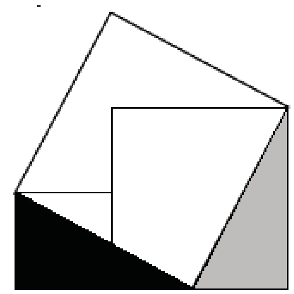
A company has designed a new logo using overlapping squares.



1. How many squares do you see in the logo? ________________________

   Describe where you see the squares.



2. The logo designer colored two triangles in the logo

   How are the two triangles related?
   Justify your answer.

3. What are the relationships between the sizes of the three squares in the original logo? Explain your findings.

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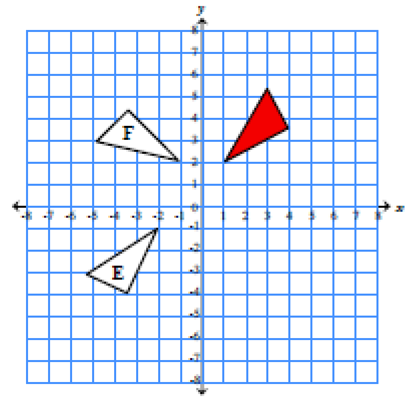
G.CO.2 and G.CO.3:



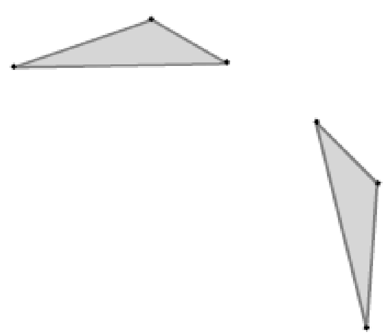
1. Draw the shaded triangle after:
   a. It has been translated -7 horizontally and $+1$ vertically. Label your answer A.
   b. It has been reflected over the x-axis. Label your answer B.
   c. It has been rotated 90° clockwise around the origin. Label your answer C.
   d. It has been reflected over the line $y = x$. Label your answer D.
2. Describe fully the single transformation that:
   a. Takes the shaded triangle onto the triangle labeled E.
   b. Takes the shaded triangle onto the triangle labeled F.

G.CO.5 – Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Resources: Students may use geometry software and/or manipulatives to model transformations and demonstrate a sequence of transformations that will carry a given figure onto another.

Example:

- The triangle in the upper left of the figure below has been reflected across a line into the triangle in the lower right of the figure. Use a straightedge and compass to construct the line across which the triangle was reflected.

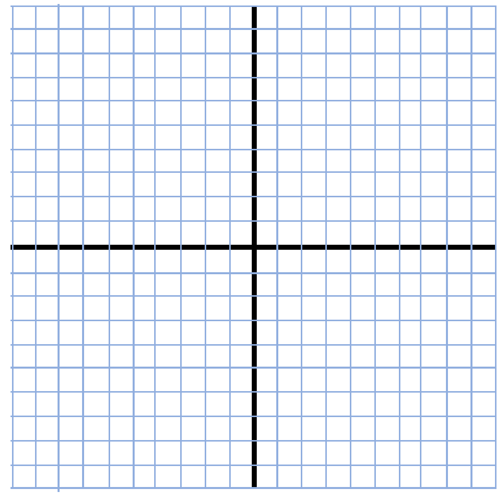


G.CO.6 – Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

Explanations: A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.

Resources: Students may use geometric software to explore the effects of rigid motion on a figure(s).

Example: Composition of a Translation and a Rotation



- ΔABC has vertices $A(-1, 0)$, $B(4, 0)$, $C(2, 6)$
  a. Draw ΔABC on the coordinate grid provided.
  b. Translate ΔABC using the rule $(x, y) \rightarrow (x - 6, y - 5)$ to create $\Delta A'B'C'$. Record the new coordinate grid (using a different color if possible).

     A'_____________ B'_____________ C'_____________
  c. Rotate $\Delta A'B'C'$ 90°CCW using the rule $(x, y) \rightarrow$________ to create $\Delta A"B"C"$. Record the new coordinates below and add the triangle to your coordinate grid (using a different color if possible).

     $A"$_____________ $B"$_____________ $C"$_____________
  d. Write ONE rule below that would change ΔABC to $\Delta A"B"C"$ in one step.

     ____

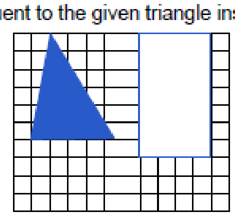
G.CO.7 – Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Explanations:

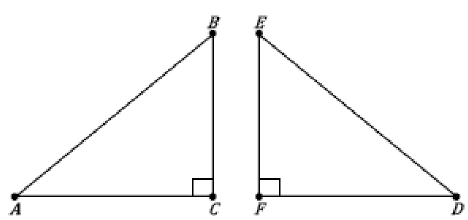
- A rigid motion is a transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are assumed to preserve distances and angle measures.
- Congruence of triangles – Two triangles are said to be congruent if one can be exactly superimposed on the other by a rigid motion, and the congruence theorems specify the conditions under which this can occur.

Example:

- How many ways can you construct a triangle congruent to the given triangle inside the rectangle? Demonstrate each.



G.CO.8 – Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.



Example:

- Josh is told that two triangles ABC and DEF share two sets of congruent sides and one pair of congruent angles: AB is congruent to DE, BC is congruent to EF, and angle C is congruent to angle F. He is asked if these two triangles must be congruent. Josh draws the two triangles below and says, "They are definitely congruent because they share all three side lengths"!
  - Explain Josh's reasoning using one of the triangle congruence criteria: ASA, SSS, SAS.
  - Give an example of two triangles ABC and DEF, fitting the criteria of this problem, which are not congruent.

G.CO.10 – Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*

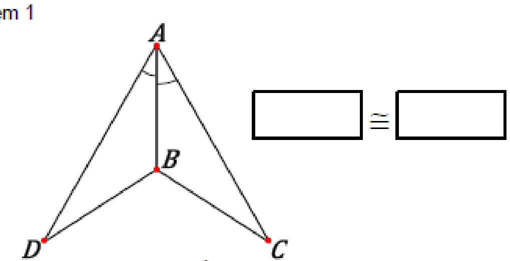
Resources: Students may use geometric simulations (computer software or graphing calculator) to explore theorems about triangles.

Example:

- For items 1 and 2, what additional information is required in order to prove the two triangles are congruent using the provided justification?
- Use the set of choices in the box below. Select a side or angle and place it in the appropriate region. Only one side or angle can be placed in each region.

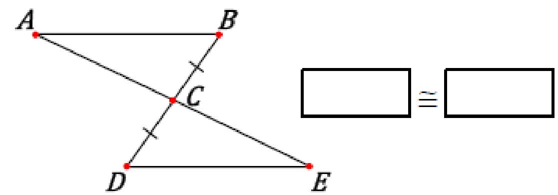
$\overline{AB}$	$\overline{AC}$	$\overline{AD}$	$\overline{BC}$
$\overline{BD}$	$\overline{CD}$	$\overline{CE}$	$\overline{DE}$
$\angle ABC$	$\angle ABD$	$\angle ACB$	$\angle ADB$
$\angle BAC$	$\angle CDE$	$\angle CED$	$\angle DCE$

Item 1



ASA Postulate

Item 2



SAS Theorem

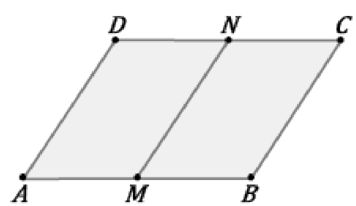
Key: Item 1 – $\angle ABD \cong \angle ABC$ Item 2 – $AC \cong CE$

G.CO.11 – Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Resources: Students may use geometric simulations (computer software or graphing calculator) to explore theorems about parallelograms.

Example:

- Suppose that $ABCD$ is a parallelogram, and that M and N are the midpoints of $\overline{AB}$ and $\overline{CD}$, respectively. Prove that $MN = AD$, and that the line $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{AD}$.

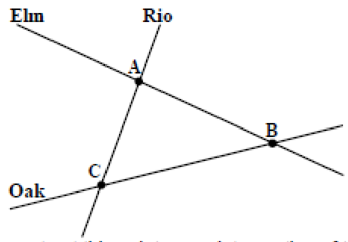


G.CO.12 – Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

Resources: Students may use geometric software to make geometric constructions.

Examples:

- Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.
- Construct the circumventer of a given triangle.
- You have been asked to place a warehouse so that it is an equal distance from the three roads indicated on the following map. Find this location and show your work.



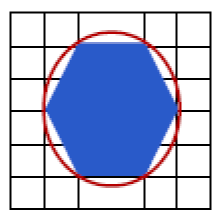
- Show how to fold your paper to physically construct this point as an intersection of two creases.
- Explain why the above construction works and, in particular, why you only needed to make two creases.

G.CO.13 – Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Resources: Students may use geometric software to make geometric constructions.

Example:

- Find two ways to construct a hexagon inscribed in a circle as shown.



Differentiation Strategies and Activities

Use interactive Applets (see ** above) to appeal to the visual learner.

Activities

1. Introducing Translations

http://www.regentsprep.org/regents/math/geometry/GT2/Activity.htm

Follow the directions on the page for both translations and reflections.

2. Introducing Other Transformations:

- Use projector and utilize website http://www.misterteacher.com/alphabetgeometry/reflection.html
- Discuss the type of transformation to be 'discovered' today.

- Use the “Practice” with the grid turned on (bottom icon on right)
- Choose a shape and transformation.
- Have students sketch the chosen figure on graph paper. Then have them sketch how/where they believe the figure will change/move.
- Enact transformation on the website.

3. Introducing Vertical angles:

- Use projector to utilize the website: http://www.mathwarehouse.com/geometry/angle/interactive-vertical-angles.php
- Manipulate the vertical angles to demonstrate the congruent angles regardless of the shape the X makes.
- Have students hypothesize the measure of the other two angles.

4. Introducing 2 Parallel lines cut by a transversal (Labeling/Understanding locations)

- Make up 2 sets of cards- 3x5 index cards cut in half work well
- Label one set in red lettering with pairs labeled “Corresponding 1”, “Corresponding 2”, etc
- Label on set in blue lettering with pairs labeled “Alt. Int. 1”, “Alt. Int. 2”, “Alt. Ext. 1” &”Alt. Ext. 2”
- Use electrical tape to make a set of the 3 lines on the floor for each group of students.
- Have students “label” their diagram with the red set first.
- Walk around to check correctness.
- Do the same with the blue set.
- Alternate students and colors to ensure each student practices with each color set.

5. Introducing Triangle Sum Theorem

    - Give each group 3 different types of triangles made from construction paper.
    - Have students label the 3 angles as A, B, and C.
    - Have students rip the triangles into 3 pieces-each piece has to contain an entire angle.
    - Have students tape the 3 angles against each other by the “points” of the angles.
    - Ask students:
        - What is formed by the 3 angles when taped in this manner? (it’s a line)
        - What do we know about the measure of a straight angle or a line?

Student will be able to determine that the 3 angles of any type of triangle creates a line which is why the interior sum equals 180˚.

6. Base angles of isosceles triangle are equal.

- Give each student in the group 3 pieces of linguini, ruler and a protractor.
- Have students break 1 piece of linguini in half.
- Have student break the 2^{nd} linguini into 3 pieces of differing sizes.
- Have student make an isosceles triangle using 1 of the 3 base pieces. Have students measure the base angles & record results.
- Have students make 2 more triangles using remaining 2 base lengths & repeat procedure.
- Have students create a rule about their findings.

7. Introducing Triangle Medians

- Give each student their own triangle made of construction paper.

- Have students label the 3 angles as A, B, and C.
- Have student fold the triangle in half along segment AB. Instruct them to line up endpoints carefully before making the crease.
- Have students repeat the process for BC and AC.
- Have students describe what they notice about the 3 medians.
- Have students lightly draw lines using pencil in the 3 creases. What do they notice occurred?

Differentiation Strategies:

- All students should be evaluated on the basic mathematical requirements of each topic.
- Most students should be evaluated on the basic algebraic requirements of each topic.
- Some students should be evaluated on the complex (multi-step) algebraic requirements of each topic.

As an example using 2 lines cut by a transversal:

- All students should be able to determine the other 7 angle measures when given 1 angle measure.
- Most students should be able to solve for x, given a variable expression for 1 angle measure and the measure of another angle.
- Some students should be able to solve for x and give the angle measure when given variable expressions for 2 different angles.

Differentiation Strategies for Special Education Students
Differentiation Strategies for Gifted and Talented Students
Differentiation Strategies for ELL Students
Differentiation Strategies for At Risk Students

Honors:

The students enrolled in this level will be assigned questions of greater complexity

Standard	Fundamental	Average	Advanced
G-CO.A.1 G-CO.A.2	**Naming points, lines, and planes.**	**Describe the difference between congruence and equal**	**Construct angle bisectors and perpendicular bisectors**
G-CO.A.3 - 5 G-CO.B.6 - 7	**Identify rigid motion.**	**Compose translations**	**Find compositions of isometries including glide reflections**
G-CO.A.3 - 5 G-CO.B.6 - 7	**Find a scale factor**	**Draw transformations using compositions of rigid transformations and dilations.**	**Use transformations to determine whether figures are similar.**
G-CO.B.8 G-CO.C.9 - 11	**Find and use patterns.**	**Use inductive reasoning**	**Collect information to make a conjecture**

G-CO.B.8 G-CO.C.9 - 11	Find and use congruent parts.	Prove triangles are congruent using SSS, SAS, ASA, AAS, and HL.	Write proofs using CPCTC
G.CO.D.12	Identify solids from a net.	Draw a solid from a net	Identify isometric drawings and orthographic drawings
G.CO.D.13	Use the isosceles triangle theorem to find missing angle measures.	Use Algebra to find values of all triangles especially isosceles triangles	Write a proof for the isosceles triangles theorem converse

Resources

- Teacher Edition Textbook
- Online Resources through McDougal Littell Classzone
- Real world Applications of Geometry
- KahnAcademy.com
- KUTA
- Edhelper.com
- Explore Learning
- Miras
- Protractors
- Geometer's Sketchpad

Online resources listed above in "Activities"

Wayne School District
Curriculum

Content Area/ Grade Level/ Course:	**Mathematics 9 - 12** **Geometry**
Unit Plan Title:	**Unit 3: Similarity, Right Triangles, and Trigonometry**
Time Frame	**7 weeks**

Anchor Standards/Domain

G-SRT Similarity, Right Triangles, and Trigonometry
G-MG Modeling with Geometry

Unit Summary

Students apply their earlier experience with dilations and proportional reasoning to build formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean theorem. Students develop the Laws of Sines and Cosines in order to find missing measure of general triangles. They are able to distinguish whether three given measure (angles or sides) define 0, 1, 2, or infinitely many triangles.

Standard Number(s)

G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G-SRT.B.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G-SRT.D.9 (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.

8.1.12.CS.4 - Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1 - Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5 - Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.DA.6 - Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.

9.4.12.CI.1 - Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).

9.4.12.CT.2 - Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)

9.4.12.TL.2 - Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.IML.3 - Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

RST.9-10.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

RST.9-10.5. Analyze the relationships among concepts in a text, including relationships among key terms (e.g.,force,, friction, reaction force, energy).

RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

WHST.9-10.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

WHST.9-10.5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.

WHST.9-10.6. Use technology , including the Internet, to produce, share, and update writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

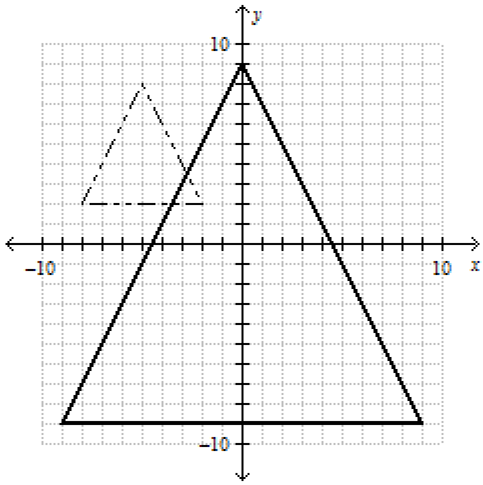
Essential Question(s)

- How do you change a figure's size without changing its shape?
- How do you identify corresponding parts of similar triangles?
- How do you find a side length or angle measure in a right triangle?
- How do trigonometric ratios relate to similar right triangles?
- How do you solve problems that involve measurements of triangles?

EXEMPLARS

G-SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor:
a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

The dashed triangle is the image of the solid triangle for a dilation with center at the origin. What is the scale factor?



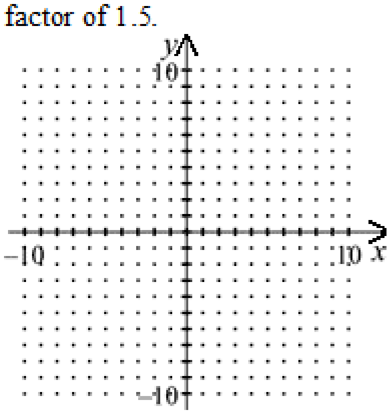
a. $\frac{2}{3}$

b. $\frac{3}{2}$

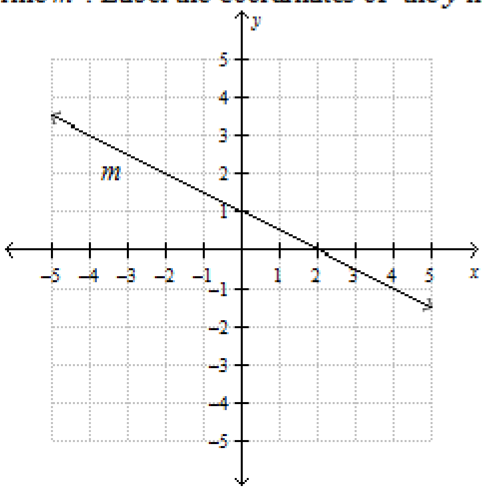
c. 3

d. $\frac{1}{3}$

Graph $\overline{DE}$ with D (–3, 4) and E (–2, –1). Then graph its dilation using the origin as the center and a scale factor of 1.5.



Part A: Draw a dilation of m with the origin as the center of dilation and a scale factor of $\frac{1}{2}$. Label the dilated line m'. Label the coordinates of the y-intercept.



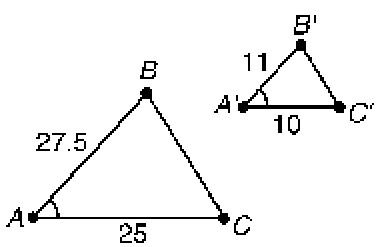
Part B: Draw a dilation of m with the origin as the center of dilation and a scale factor of 2.5, to m. Label this line m'', and label the y-intercept with its coordinates.
Part C: Draw a conclusion based on the answers to ***Parts A*** and ***B***. What is the relationship between the three lines, m, m', and m''?

G-SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

If two polygons are SIMILAR, then the corresponding sides must be _____.

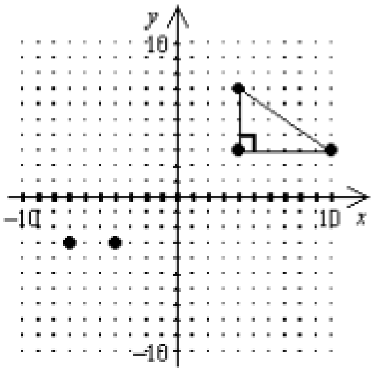
a. proportional
b. congruent
c. parallel
d. similar

This drawing illustrates ______.



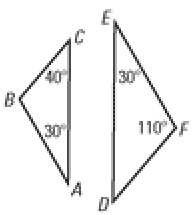
a. AA Similarity
b. SAS Congruence
c. SSS Similarity
d. SAS Similarity

One triangle is shown on the coordinate grid. Two vertices for a second triangle are also shown. Which coordinates for the third vertex will form another triangle that is similar to the triangle that is shown?

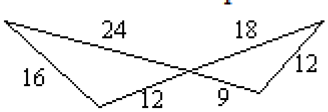


a. $(-7, -7)$ b. $(-7, -2)$ c. $(-7, -5)$ d. $(-7, 5)$

Determine whether the triangles are similar. If the are, write a similarity statement.



Tell whether each pair of triangles is similar. Explain your reasoning.

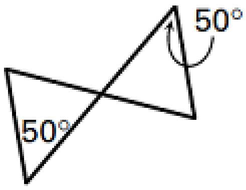


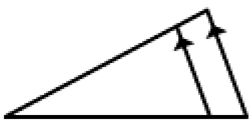
Standing next to each other, a woman casts a 41.4-inch shadow and her 40-inch-tall son casts a 24-inch shadow. What is the height of the woman to the nearest inch?

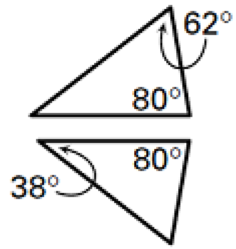
A photo needs to be enlarged from an original with a length of 7 inches and a width of 5 inches to a size where the new width is 15 inches. What is the new length? What is the scale factor?

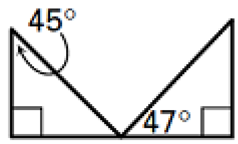
G-SRT.A.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Use the Angle-Angle Similarity Postulate to determine which pair of triangles is *not* similar.

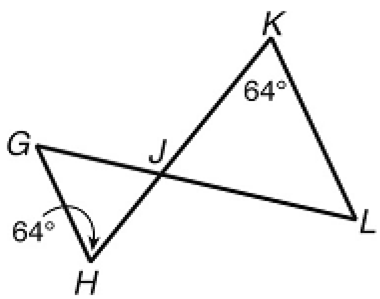
a. 

b. 

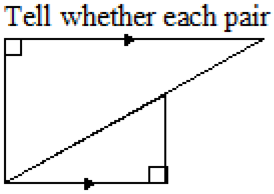
c. 

d. 

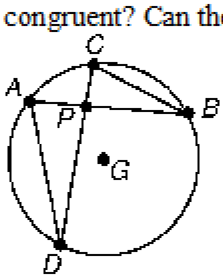
Explain why the triangles are similar and write a similarity statement.



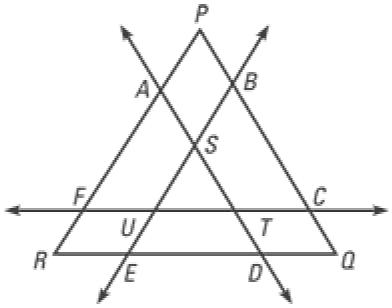
Tell whether each pair of triangles is similar. Explain your reasoning.



Given: $\odot G$ with intersecting chords $\overline{AB}$ and $\overline{CD}$ that meet at P. Can triangles ΔADP and ΔCBP be congruent? Can they be similar? Can they be neither congruent nor similar? Explain.

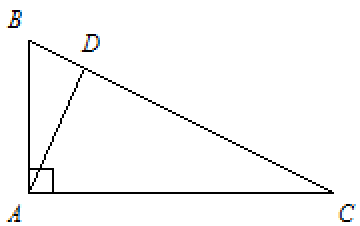


$\overleftrightarrow{UT} \parallel \overline{RQ}$, $\overleftrightarrow{SU} \parallel \overline{PR}$, and $\overleftrightarrow{ST} \parallel \overline{PQ}$. Explain how you can prove that $\Delta PQR \sim \Delta STU$.



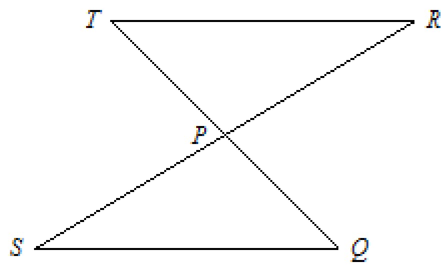
G-SRT.B.4 Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*

A diagram from a proof of the Pythagorean Theorem is shown. Which statement would NOT be used in the proof?



a. $(AB)^2 + (AC)^2 = (BC)[(BD) + (DC)] \Rightarrow (AB)^2 + (AC)^2 = (BC)$
b. $\Delta BAC \sim \Delta BDA \sim \Delta ADC$
c. $\dfrac{AB}{BC} = \dfrac{BD}{AB}$ and $\dfrac{AC}{BC} = \dfrac{DC}{AC}$
d. ΔABC is a right triangle with an altitude $\overline{AD}$.

Given: P is the midpoint of $\overline{TQ}$ and $\overline{RS}$.
Prove: $\Delta TPR \cong \Delta QPS$



Complete the proof.

Proof:

Statements	Reasons
1. P is the midpoint of $\overline{TQ}$ and $\overline{RS}$.	1. Given
2. $\overline{TP} \cong \overline{QP}$, $\overline{RP} \cong \overline{SP}$	2. [1]
3. [2]	3. Vertical Angles Theorem
4. $\Delta TPR \cong \Delta QPS$	4. [3]

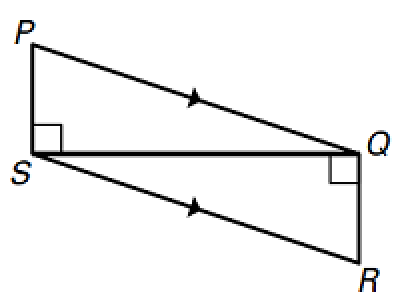
a. [1]. Definition of midpoint
[2] $\angle TPR \cong \angle QPS$
[3] SAS

b. [1] Definition of midpoint
[2] $\overline{RT} \cong \overline{SQ}$
[3] SSS

c. [1] Definition of midpoint
[2] $\angle PRT \cong \angle PSQ$
[3] SAS

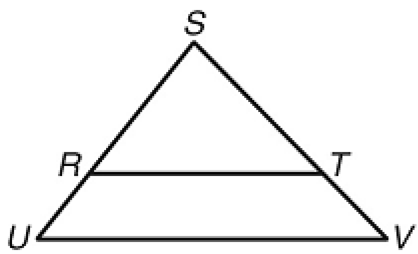
d. [1] Definition of midpoint
[2] $\angle TPR \cong \angle QPS$
[3] SSS

Decide whether enough information is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate or theorem you would use.



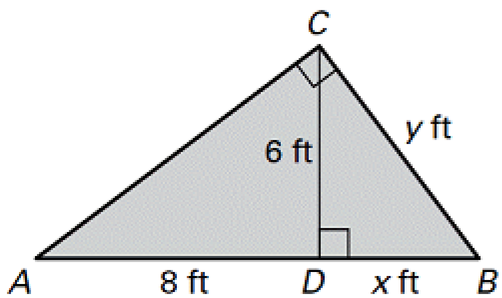
Given: $A(2,1)$, $B(6,3)$, $C(8,1)$, $D(4,2)$, and $E(5,1)$
Prove: $\triangle ABC \sim \triangle ADE$

Given: $SR = 2RU$ and $ST = 2TV$.
Prove: $\triangle USV \sim \triangle RST$



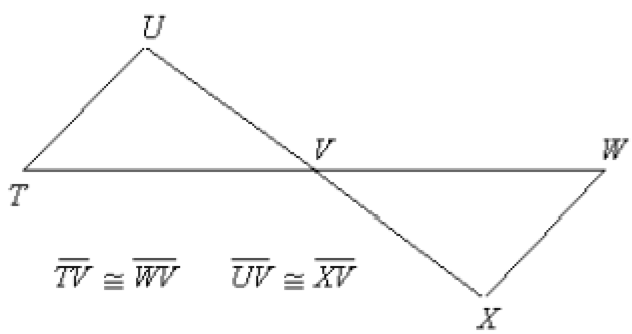
G-SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

What is the value of x?



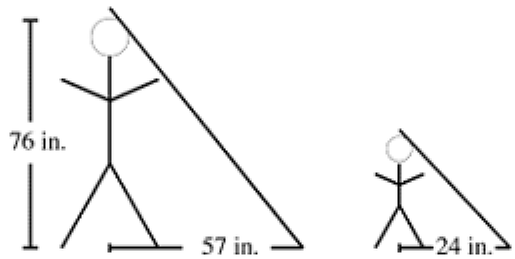
a. 4.5
b. 7.5
c. 10
d. 10.5
e. 12.5

Refer to the figure shown. Which of the following statements is true?



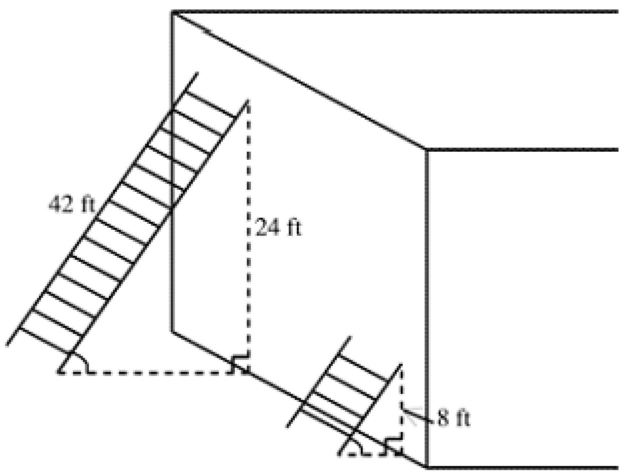
a. $\Delta TUV \cong \Delta XWV$ by *ASA*
b. $\Delta TUV \cong \Delta VWX$ by SAS
c. $\Delta TUV \cong \Delta WXV$ by *SAS*
d. $\Delta TUV \cong \Delta WXV$ by SSS

At the same time of day, a man who is 76 inches tall casts a 57-inch shadow and his son casts a 24-inch shadow. What is the height of the man's son? (Figures may not be drawn to scale.)



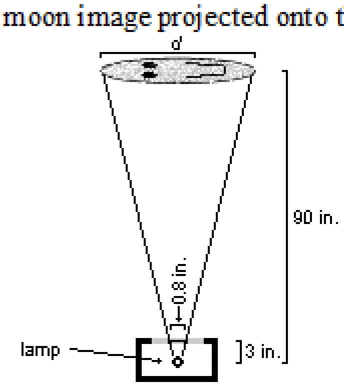
a. 33 in. b. 32 in. c. 81 in. d. 108 in.

The triangles formed by two ladders leaning against a wall are similar. How long is the shorter ladder?



a. 6 ft b. 14 ft c. 28 ft d. 10 ft

A "soother" toy for a baby projects images onto the ceiling.
Using the information in the diagram, find the scale factor of the dilation. Then calculate the diameter of the moon image projected onto the ceiling.



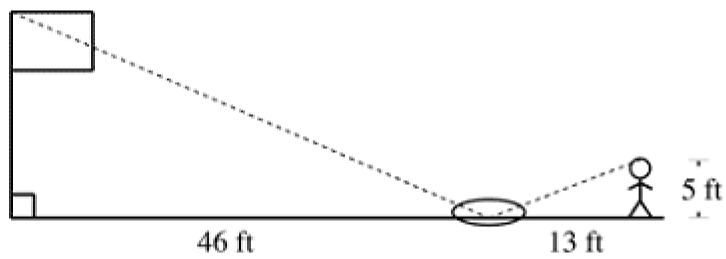
a. Scale factor = 30; d = 24 in.

b. Scale factor = 112.5; d = 337.5 in.

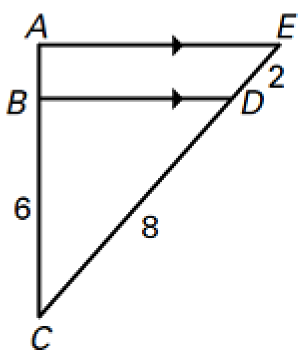
c. Scale factor = $\frac{4}{15}$; d = 24 in.

d. Scale factor = $\frac{15}{4}$; d = 337.5 in.

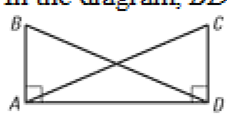
Karen wanted to measure the height of her school's flagpole. She placed a mirror on the ground 46 feet from the flagpole, and then walked backwards until she was able to see the top of the pole in the mirror. Her eyes were 5 feet above the ground and she was 13 feet from the mirror. Using similar triangles, find the height of the flagpole to the nearest hundredth of a foot. (Figures may not be drawn to scale.)



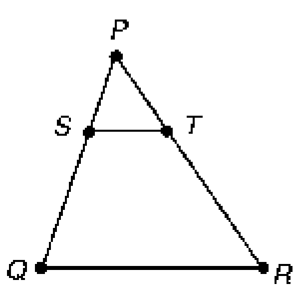
Find the length of $\overline{AB}$.



In the diagram, $\overline{BD} \cong \overline{AC}$. Explain why $\overline{AB} \cong \overline{CD}$.



Given that $\Delta PQR \sim \Delta PST$, explain why $\overline{QR} \parallel \overline{ST}$.



G-SRT.C.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

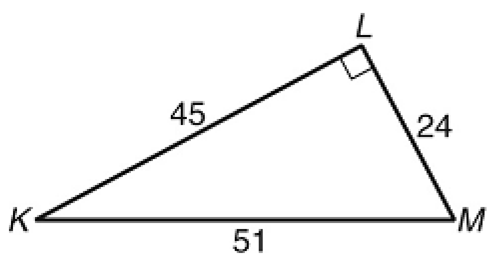
The hypotenuse of a 30°-60°-90° triangle measures $10\sqrt{3}$ inches. What is the measure of the longer leg?

a. 5 in.
b. $5\sqrt{3}$ in.
c. 10 in.
d. 15 in.

One leg of a 45°-45°-90° triangle measures 12 centimeters. What is the length of the hypotenuse?

a. $4\sqrt{3}$ cm
b. $6\sqrt{2}$ cm
c. $12\sqrt{2}$ cm
d. $12\sqrt{3}$ cm

What is tan *K*?



a. $\frac{8}{17}$
b. $\frac{8}{15}$
c. $\frac{15}{17}$
d. $\frac{15}{8}$

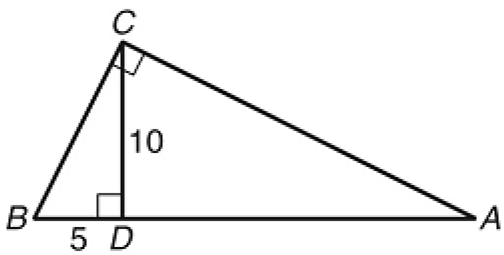
Use a special right triangle to write tan 60° as a fraction.

a. $\frac{\sqrt{3}}{1}$
b. $\frac{1}{\sqrt{3}}$
c. $\frac{\sqrt{2}}{1}$
d. $\frac{\sqrt{3}}{2}$

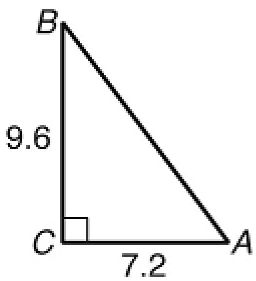
A skateboard ramp has a slope of $\frac{2}{5}$. Which is the angle the ramp makes with the ground?

a. 22°
b. 24°
c. 66°
d. Not here

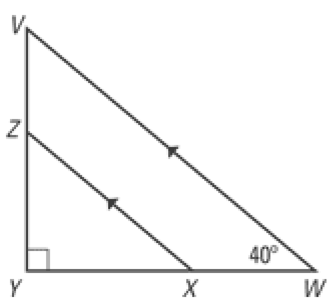
Find AB. If necessary, give the answer in simplest radical form.



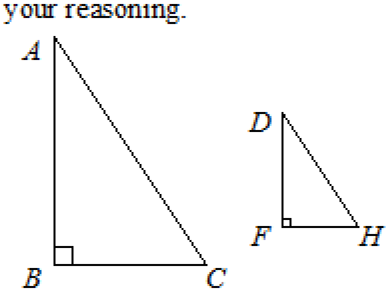
To the nearest degree what is $m\angle A$?



In the diagram, $\overline{VW} \parallel \overline{ZX}$. If $YX = 5$, what is ZX? Explain how you got your answer.



Triangle ABC and triangle DFH are similar. If $\sin A = 0.4$ and $DH = 3$, find FH. Show your work or explain your reasoning.

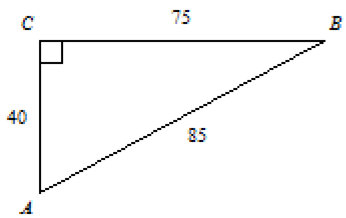


G-SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

Find two angles that satisfy the equation $\sin(4x + 14)° = \cos(-3x + 73)°$.

a. 26°; 74°
b. 29°; 61°
c. 64°; 116°
d. 26°; 64°

$\angle A$ and $\angle B$ are complementary angles as shown in right triangle ABC. Find the sine of $\angle A$ and the cosine of $\angle B$. Then describe how they are related.



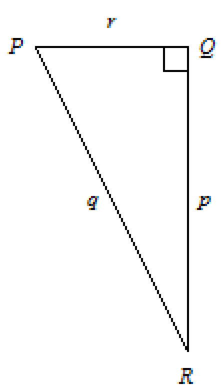
a. $\sin A = \cos B = \frac{8}{17}$; they are the same ratio

b. $\sin A = \cos B = \frac{17}{15}$; they are the same ratio

c. $\sin A = \cos B = \frac{15}{8}$; they are the same ratio

d. $\sin A = \cos B = \frac{15}{17}$; they are the same ratio

$\angle P$ and $\angle Q$ are complementary angles as shown in right triangle PQR. Write ratios, in terms of p, q, and r, for sin R and cos P. Then describe how they are related.



a. $\sin R = \frac{q}{r}$, $\cos P = \frac{q}{p}$; they are the same ratio

b. $\sin R = \frac{r}{q}$, $\cos P = \frac{r}{q}$; they are the same ratio

c. $\sin R = \frac{r}{q}$, $\cos P = \frac{p}{q}$; they are different ratios

d. $\sin R = \frac{p}{q}$, $\cos P = \frac{p}{q}$; they are the same ratio

Find the values of the other five trigonometric functions of θ, given that $\cos\theta = \frac{5}{13}$, $0 < \theta < \frac{\pi}{2}$.

Find the values of the other five trigonometric functions of θ, given that $\sec\theta = \frac{5}{3}$, $\frac{3\pi}{2} < \theta < 2\pi$.

Simplify the expression.

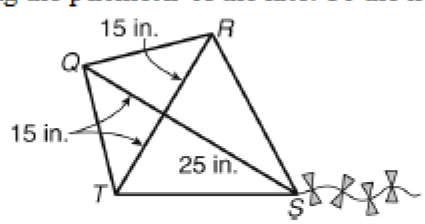
$$\sin^2\left(\frac{\pi}{2} - \theta\right) + \cos^2\left(\frac{\pi}{2} - \theta\right)$$

G-SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

. A 12-foot ladder is leaning up against the side of a house. The ladder makes an angle of 62° with the ground. How far up the side of the house does the ladder reach?

a. 0.1 foot
b. 5.6 feet
c. 10.6 feet
d. 13.6 feet
e. 25.6 feet

A kite frame consists of two pieces of wood placed along the diagonals. Decorative binding will be placed along the perimeter of the kite. To the nearest tenth of an inch, how much binding is needed?



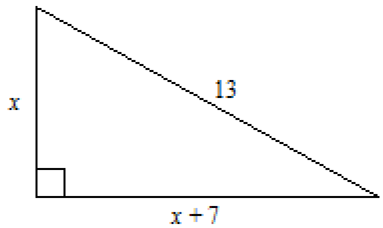
a. 70.0 in.
b. 90.4 in.
c. 100.7 in.
d. 140.0 in.

A helicopter pilot sights a landmark at an angle of depression of 22°. The altitude of the helicopter is 1450 feet. To the nearest foot, what is the horizontal distance from the helicopter to the landmark.

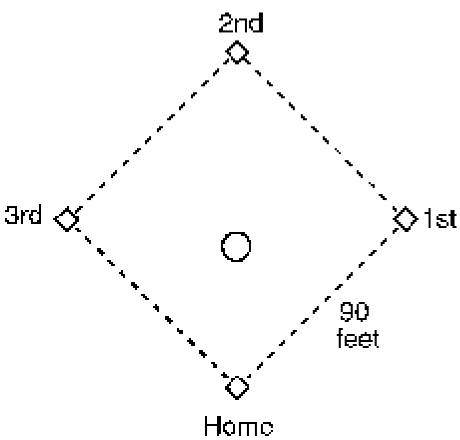
a. 543 ft
b. 586 ft
c. 3589 ft
d. 3871 ft

The angle of elevation from the tip of a flagpole's shadow to the top o the flagpole is 63°. The length of the shadow is about 12 feet. How tall is the flagpole to the nearest tenth of a foot?

The hypotenuse of a right triangle is 13 cm. One of the legs is 7 cm longer than the other leg. Find the area of the triangle. (Hint: Use the Pythagorean Theorem.)



A baseball "diamond" is a square of side length 90 feet. How far is the throw, to one decimal place, from home plate to second base?



G-SRT.D.9 (+) Derive the formula $A = 1/2\ ab\ \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Triangle ABC is not a right triangle. Side lengths a and b are given, along with the measure of $\angle C$, the angle between them. Write a formula for the area of the triangle in terms of the given quantities.

a. $A = ab \sin C$

b. $A = \frac{1}{2} ab \cos C$

c. $A = \frac{1}{2} ab$

d. $A = \frac{1}{2} ab \sin C$

A triangle has a side with length 6 feet and another side with length 8 feet. The angle between the sides measures 73°. Find the area of the triangle. Round your answer to the nearest tenth.

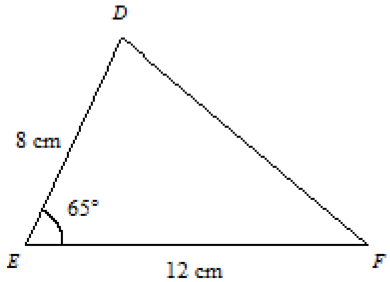
a. 1752.0 ft^2

b. 7.0 ft^2

c. 45.9 ft^2

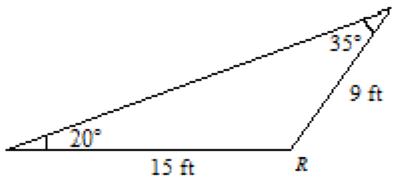
d. 23.0 ft^2

Determine the area of $\triangle DEF$. Round your answer to the nearest tenth of a cm^2.



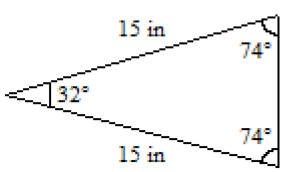
a. $20.3\ cm^2$
b. $87.0\ cm^2$
c. $43.5\ cm^2$
d. $14.1\ cm^2$

Jason and Melissa have a triangular patio they want to cover with indoor-outdoor carpeting. The dimensions of the patio are shown. If the carpeting sells for $12.50 per square yard, and the carpeting is only sold in full square yards, how much will it cost them to cover their patio?



a. $112.50
b. $75
c. $87.50
d. $100

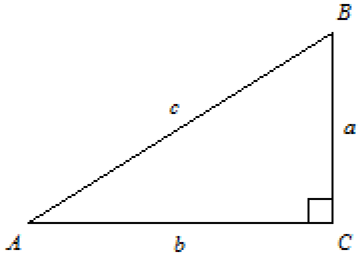
A high school pennant is in the shape of an isosceles triangle. Determine the area of the pennant to the nearest tenth of an in^2.



a. $59.6\ in^2$
b. $29.8\ in^2$
c. $95.4\ in^2$
d. $108.1\ in^2$

State in words how to determine the area of any triangle if two side lengths are given, along with the measure of the angle between them. (Do not use math symbols in your explanation.)

Triangle ABC is a right triangle.



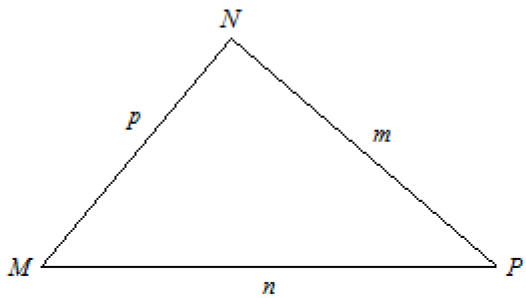
Part A: Write a formula for the area of $\triangle ABC$ using $\overline{AC}$ as the base and $\overline{BC}$ as the height.

Part B: Write a formula for the area of $\triangle ABC$ using $\overline{AC}$, $\overline{BC}$ and the included angle.

Part C: Compare the results from ***Parts A*** and ***B*** to determine the value of sin 90°. Explain your reasoning.

G-SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

Solve the triangle. $m\angle N = 118°$, $m\angle P = 33°$, and $m = 15$. Round to the nearest tenth.



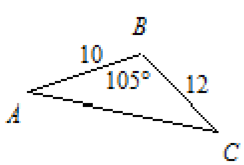
a. $m\angle M = 29°$, $n \approx 27.3$, $p \approx 13.4$

b. $m\angle M = 29°$, $n \approx 8.2$, $p \approx 13.4$

c. $m\angle M = 29°$, $n \approx 27.3$, $p \approx 16.9$

d. $m\angle M = 29°$, $n \approx 8.2$, $p \approx 16.9$

Find AC. Round to the nearest tenth.



a. $AC = 17.5$

b. $AC = 306.1$

c. $AC = 16.6$

d. $AC = 10.3$

Solve triangle ABC given that $A = 45°$, $B = 54°$, and $b = 70$.

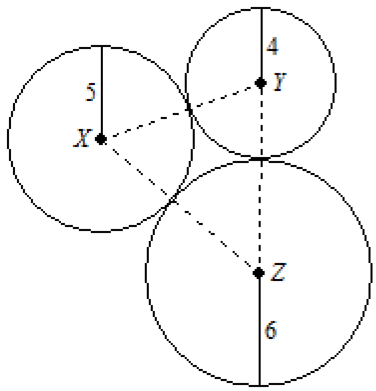
a. $C = 81°$, $a = 80.09$, $c = 97.78$

b. $C = 261°$, $a = 61.18$, $c = 85.46$

c. $C = 81°$, $a = 61.18$, $c = 85.46$

d. $C = 261°$, $a = 80.09$, $c = 97.78$

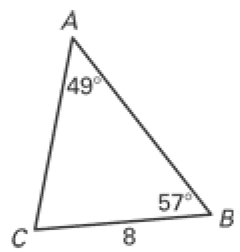
Three circular disks are placed next to each other as shown. The disks have radii of 4 cm, 5 cm, and 6 cm. The centers of the disks form ΔYXZ. Find $m\angle YXZ$ to the nearest degree.



a. $m\angle YXZ = 65°$
b. $m\angle YXZ = 90°$
c. $m\angle YXZ = 59°$
d. $m\angle YXZ = 51°$

Solve ΔABC. $C = 150°$, $a = 15$, $b = 14$

Solve ΔABC.

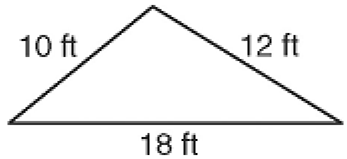


G-SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Two airplanes leave the airport at the same time. One airplane flies due east at a speed of 300 miles per hour. The other airplane flies east-northeast at a speed of 350 miles per hour (the angle between the two directions is 22.5°). If the planes are at the same altitude, how far apart are they after 2 hours? Round your answer to the nearest mile.

a. 136 miles
b. 272 miles
c. 100 miles
d. 727 miles

The Jackson family wants to cover a section of a wall with fabric. The wall has the dimensions shown below. The fabric they want to use costs $4.50 per square foot. What is the approximate cost of covering this area? (*Picture is not to scale.*)

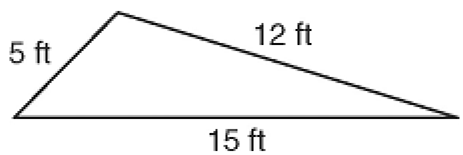


a. $56.57
b. $119.65
c. $191.06
d. $254.56

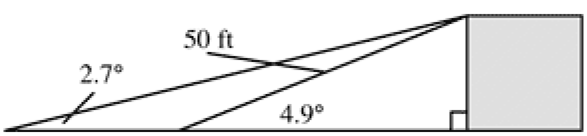
Island A is 220 miles from island B. A ship captain travels 310 miles from island A and then finds that he is off course and 170 miles from island B. What angle, in degrees, must he turn through to head straight for island B? Round the answer to two decimal places. (Hint: Be careful to properly identify which angle is the turning angle.)

a. 46.62°
b. 43.38°
c. 136.62°
d. 93.23°

The Sarao family is installing a pond in their backyard with the dimensions shown below. They want to stock the pond with fish that require 2 square feet of surface area each. How many fish can they put in their new pond? Round your answer down to the nearest whole fish. (*Picture is not to scale.*)

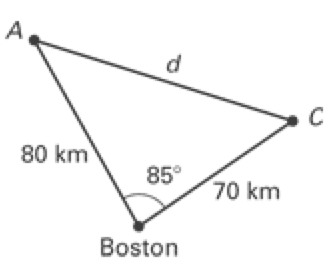


A 50-foot ramp makes an angle of 4.9° with the horizontal. To meet new accessibility guidelines, a new ramp must be built so it makes an angle of 2.7° with the horizontal.



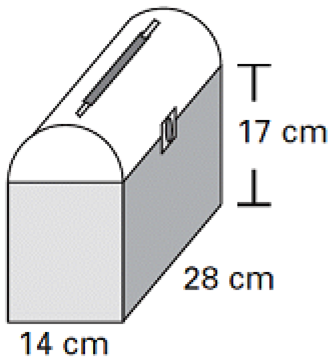
What will be the length of the new ramp?

Two ships leave Boston Harbor at the same time. What is the distance between ships A and C after they have traveled 80 kilometers and 70 kilometers respectively?



G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

A lunch box consists of half of a cylinder placed on top of a rectangular prism. What the is the volume of the lunch box?



a. 2155 cm^3

b. 4310 cm^3

c. 8819 cm^3

d. 10,974 cm^3

A pup tent has a rectangular floor, two rectangular sides, and two triangular sides. Which geometric shape best models a pup tent?

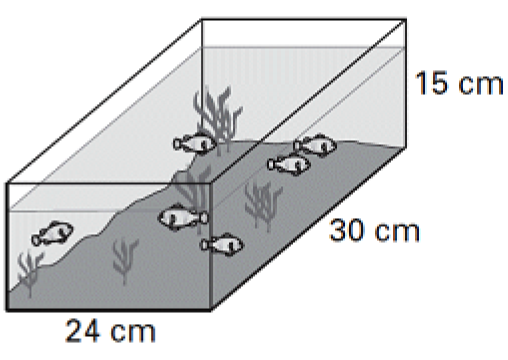
a. rectangular prism

b. rectangular pyramid

c. triangular prism

d. triangular pyramid

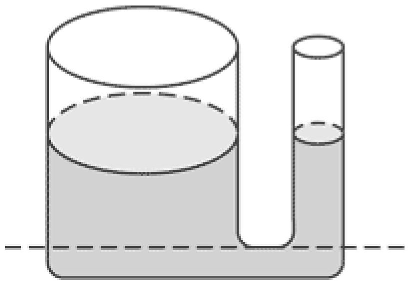
The weight of 1 cm^3 of water is exactly 1 g. A small desk top fish tank is pictured below. How much does the water in the tank weigh in kilograms?



A right rectangular tank with a 12 in. by 8 in. base is filled with water to a depth of 5 in. If the water rises $\frac{2}{3}$ in. when a solid cube is completely submerged in the tank, find the length of an edge of the cube.

A water tank has a depth gauge attached to it as shown. The depth of the water in the gauge matches the depth of the water in the tank. Suppose the tank's radius is 10 in. and the gauge's radius is 0.5 in.

Part A: Find the ratio of the volume of water in the tank to the volume of water in the gauge, above the dotted line. Assume both containers are cylindrical.
Part B: Find the ratio of the lateral areas of the two volumes of water.



Not drawn to scale

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

A block of gold with a mass of 9,660 kilograms has a volume of 0.5 cubic meters. What is the density of gold?

a. 28,980 kg/m³	c. 4,830 kg/m³
b. 19,320 kg/m³	d. 9,660.5 kg/m³

The volume of a ball is 768 cm³ and its density is 3.2 g/cm³. What is the mass of the ball? Round to the nearest tenth if necessary.

a. 240 g	c. 2,457.6 g
b. 24.6 g	d. 4.2 g

An elevator building code requires at least 2.2 square feet per person. The depth of an elevator box is 7 feet. How wide must the elevator be to legally accommodate 15 people? Round to the nearest tenth if necessary.

a. 231 ft	c. 4.8 ft
b. 3.1 ft	d. 4.7 ft

For a certain species of animal to survive, the population density must be less than 15 per square mile. In a rectangular wildlife preserve measuring 20 miles by 15 miles, scientists counted 3,740 of the animals.

Part A: Is there enough area for all the animals to survive? Explain.

Part B: If there is enough area, how many more animals could be moved to the preserve? If there is not enough area, how could you change the 15-mile width of the preserve so that the preserve is large enough? Show how you found your answer.

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.

Drake wants to reduce an 8-inch by 10-inch photo so that the width is 5 inches. What will be the measure of the length?

a. 4 in.
b. $6\frac{1}{4}$ in.
c. 7 in.
d. 16 in.

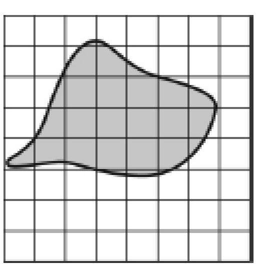
Troy is painting the U.S. Capitol Building on a museum wall. The actual building is 752 feet long and 288 feet tall. He will use the entire wall height, which is 16 feet. To the nearest half inch, how much width will he need?

a. $6\,ft\,1\frac{1}{2}\,in$
b. 29 ft
c. $41\,ft\,9\frac{1}{2}\,in.$
d. 13,536 ft

A designer of decorative tiles wants to have some custom tiles made in the shape of a rhombus, with one diagonal twice the length of the other. If each side of a tile is to be 16 inches long, what will be the length of its shorter diagonal, to the nearest hundredth of an inch?

a. 7.16 in
b. 8.00 in
c. 14.31 in
d. 28.62 in

Sod is going to be placed over an irregularly shaped area. If sod costs $6 a square yard, estimate the cost of the sod needed to cover the area. The grid has squares with side lengths of 2 feet.



Enduring Understandings

- Two geometric figures are similar when corresponding lengths are proportional and corresponding angles are congruent.

- If certain combinations of side lengths and angle measures of a right triangle are known, ratios can be used to find other side lengths and angle measures.
- Definitions establish meanings and remove possible misunderstandings. Other truths are more complex and difficult to see. It is often possible to verify complex truths by reasoning from simpler ones by using deductive reasoning.

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
x	**Global Awareness**	x	**Creativity and Innovation**
x	**Environmental Literacy**	x	**Critical Thinking and Problem Solving**
	Health Literacy		**Communication**
	Civic Literacy	x	**Collaboration**
	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

Students will…

- Verify the properties of dilations through experimentation.
- Define and use similarity in transformations to prove relationships and solve problems in geometric figures.
- Explain and use the relationship between similarity and trigonometric ratios.
- Prove triangle laws and theorems and use them to solve triangle problems.
- Apply geometric methods of similarity and trigonometry to model and solve real world problems.

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Summative Assessment

- Homework
- Quiz per standard
  Understand similarity in terms of similarity transformations (G-SRT.A.1 – G-SRT.A.3)
  Prove theorems involving similarity (G-SRT.B.4 – G-SRT.B.5)
  Define trigonometric ratios and solve problems involving right triangles (G-SRT.C.6 – G-SRT.C.8)
  Apply trigonometry to general triangles (G-SRT.D.9 – G-SRT.D.11)
  Apply geometric concepts in modeling situations (G-MG.A.1 – G-MG.A.3)
- Unit Test: Cover all topics of similarity, proof, and trigonometry.
- Unit Project: For example, a project on modeling, in which similarity and scale factor are used to model a real life situation in a smaller, more manageable context.
- Midterm Exam*
- Final Exam*
- Co-operative group explorations of appropriate topics

- Use of in-class questioning that builds on prior knowledge from previously learned material
- Use of communicator boards for immediate in-class feedback
- Use of clickers for immediate in-class feedback
- Benchmark Tests

Formative Assessments

- Observation of student work
- Use of math vocabulary
- Quizzes (written, oral, etc.)
- Student Presentations/Peer Evaluations

Teaching and Learning Activities

Activities

- Similarity and transformations
  http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=260
  Gizmos on Dilations, Reflections, Rotations, Similar Figures/Polygons, and Similarity in Right Triangles
  Demonstrations on Similarity and Transformations using Geometers SketchPad
- Right triangle trigonometry
  http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=261
  Gizmos on Trigonometric Ratios (Sine and Cosine Ratios, Tangent Ratio)

Differentiation Strategies

Standard	Geometry	Geometry A	Geometry H
G-SRT.A.1	Describe dilations by center and scale factor	Experiment with dilations using ruler and protractor	Verify properties of dilations through experimentation
G-SRT.A.2-3	Determine if figures are similar using definitions of similarity (AA)	Explain the meaning of similarity for triangles using similarity transformations	Establish the AA criterion using properties of similarity transformations
G-SRT.B.4-5	Use triangle theorems and congruence/similarity criteria to solve problems	Prove triangle theorems and use congruence/similarity criteria to solve problems	Prove triangle theorems and use congruence/similarity criteria to prove relationships
G-SRT.C.6-8	Use trig ratios to solve triangle problems	Use trig ratios and algebra to solve triangle problems	Apply trig ratios to algebra to complex problems
G-SRT.D.9-11	Use the Laws of Sines and Cosines to solve problems	Apply the Laws of Sines and Cosines to real life problems	Prove the Laws of Sines and Cosines

	G-MG.A.1-3	Model objects using scale factor and dilations	Solve problems in modeling situations	Design problems that can be modeled geometrically

Differentiation Strategies for Special Education Students
Differentiation Strategies for Gifted and Talented Students
Differentiation Strategies for ELL Students
Differentiation Strategies for At Risk Students

Honors:	The students enrolled in this level will be assigned questions of greater complexity.

Resources

- Teacher Edition: Geometry Textbook
- Geometers SketchPad
- SAS Curriculum Pathways
- Gizmos (Explore Learning)
- Online resources through McDougal Littell Classzone
- Real World Applications of Geometry: http://www.ics.uci.edu/~eppstein/geom.html
- NJSLS Mathematics: http://www.state.nj.us/education/cccs/2016/math/standards.pdf

Wayne School District
Curriculum

Content Area/ Grade Level/ Course:	**Mathematics** **9/10** **Geometry**
Unit Plan Title:	**Unit 4: Circles**
Time Frame	**3-4 weeks**

Anchor Standards/Domain

G-C: Circles

Unit Summary

In this unit, students will explore properties of circles. They will discover the relationship between a tangent and a radius, and apply the tangent radius property to solve related problems. The relationship between a chord, its perpendicular bisector, and the centre of a circle will be developed. Finally, students will develop the relationship between inscribed angles and a central angle subtended by the same arc.

The world around us is inherently geometric. The concrete and visual nature of geometry resonates with certain learning styles, and geometry's pervasiveness in our environment facilitates connecting the study of geometry to meaningful real-world situations. This is as true for circle geometry as for geometry in general. Whether determining the correct location for handles on a bucket, finding the centre of a circle in an irrigation project, or determining the length of a tangent to the earth from an orbiting satellite, properties of circles come into play. Studying circle properties increases students' logical thinking and deduction skills, and will be useful in high school mathematics in the study of analytic geometry, trigonometry and calculus.

Standard Number(s)

G-C.A.1 Prove that all circles are similar.

G-C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

G-C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G-C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.

G-C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

8.1.12.CS.4 - Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1 - Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5 - Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.DA.6 - Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.

9.4.12.CI.1 - Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).

9.4.12.CT.2 - Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)

9.4.12.TL.2 - Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.IML.3 - Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

RST.9-10.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

RST.9-10.5. Analyze the relationships among concepts in a text, including relationships among key terms (e.g.,force,, friction, reaction force, energy).

RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

WHST.9-10.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

WHST.9-10.5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.

WHST.9-10.6. Use technology , including the Internet, to produce, share, and update writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

Essential Question(s)

Students will focus on the following essential questions:

- How is geometry used as a way of looking at and thinking about our world?
- How are definitions, figures, and symbols used in the study of geometry?
- How are visual thinking skills and construction tools used in the study of geometry?
- How are the basic tools of geometry used to create complex figures and solve problems?
- What are the properties of circles and the relationships among angles, lines and line segments in and around circles?
- What are the special ratios of circles?
- What relationships exist among the measures of central and inscribed angles and arcs?
- How can the relationship between arcs and angles be used to find the length or measure of arcs?
- What is the relationship between the circumference and the diameter of a circle?
- How do we find and graph the equation of a circle?
- How do we prove circle conjectures?

Enduring Understandings

Students will understand…

- that all circles are similar.
- that central, inscribed, and circumscribed angles are related.
- that angles inscribed on a diameter are right angles.
- that the radius and the tangent to a circle are perpendicular.
- that relationships exist between inscribed, central, and circumscribed angles and their intercepted arcs.
- that there is a difference between inscribed and circumscribed triangles/quadrilaterals.
- that a tangent to a circle intersects the circle at exactly one point.
- that the measure of a central is used to find the arc length and the area of the sector.
- that the area of a sector of a circle is a portion of the area of the total circle.
- that the arc length of a circle is a portion of the circumference of the circle.
- that angles can be measured in degrees or radians.

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
	Global Awareness	T,E,A	**Creativity and Innovation**
	Environmental Literacy	T,E,A	**Critical Thinking and Problem Solving**
	Health Literacy	T,E,A	**Communication**

	Civic Literacy	T,E,A	Collaboration
X	Financial, Economic, Business, and Entrepreneurial Literacy		

Student Learning Targets/Objectives (Students will know/Students will understand)

Properties of Circles

- Identify and/or use parts of circles and segments associated with circles.
- Identify, determine, and/or use the radius, diameter, segment, and/or tangent of a circle.
- Identify, determine, and/or use the arcs, semicircles, sectors, and/or angles of a circle.
- Use chords, tangents, and secants to find missing arc measures or missing segment measures.

Coordinate Geometry and Measurement of Circles

- Use and/or develop procedures to determine or describe measures of circumference and/or area.
- Describe how a change in the linear dimension of a circle affects its circumference and area (e.g., How does changing the length of the radius of a circle affect the circumference of the circle?).
- Graph or find the equation of a circle, including any line or segment associated with circles.

Students will:

1) Know that all circles are similar.

Example:

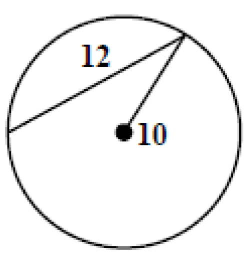
• Show the two given circles are similar by stating the necessary transformations from C to D.

C: center (2,3) radius 5; D: center (−1,4) radius 10

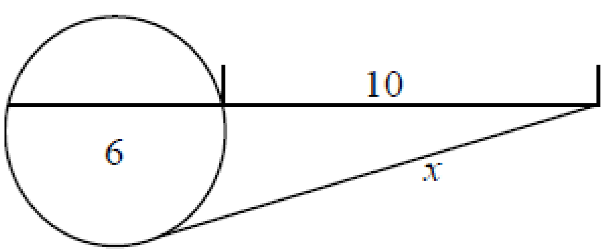
2) Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Examples:

• Given the circle below with radius of 10 and chord length of 12, find the distance from the chord to the center of the circle.



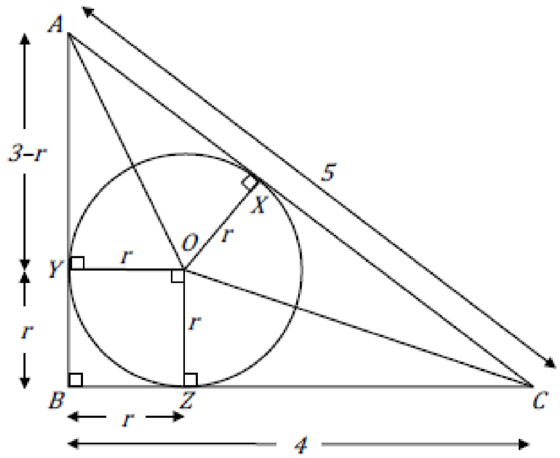
• Find the unknown length in the picture below.



3) Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

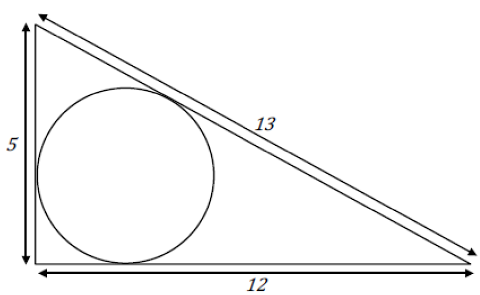
Examples:

• The following diagram shows a circle that just touches the sides of a right triangle whose sides are 3 units, 4 units, and 5 units long.

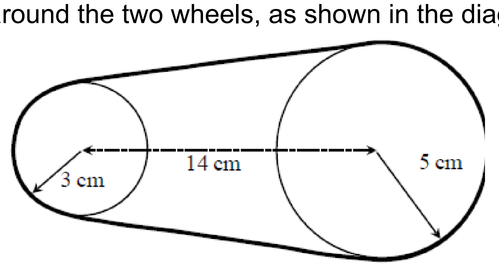


a. Explain why triangles AOX and AOY are congruent.
b. What can you say about the measures of the line segments CX and CZ?
c. Find r, the radius of the circle. Explain your work clearly and show all your calculations.

• The following diagram shows a circle that just touches the sides of a right triangle whose sides are 5 units, 12 units, and 13 units long. Draw radius lines as in the previous task and find the radius of the circle in this 5, 12, 13 right triangle. Explain your work and show your calculations.



• A certain machine is to contain two wheels, one of radius 3 centimeters and one of radius 5 centimeters, whose centers are attached to points 14 centimeters apart. The manufacturer of this machine needs to produce a belt that will fit snugly around the two wheels, as shown in the diagram below. How long should the belt be?



4) Construct a tangent line from a point outside a given circle to the circle.

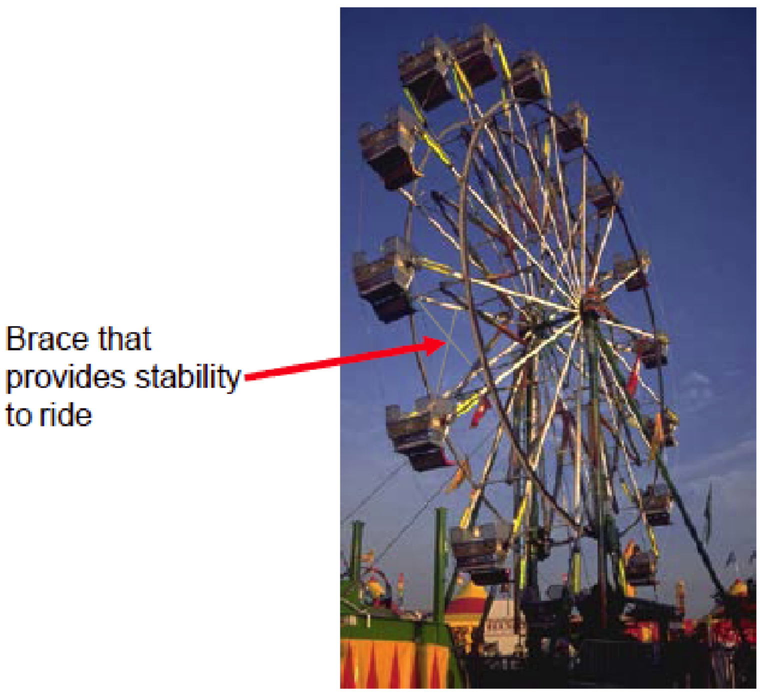
Resources: Students may use geometric simulation software to make geometric constructions.

5) Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Examples: Consider the Ferris Wheel

• The amusement park has discovered that the brace that provides stability to the Ferris wheel has been damaged and needs work. The arc length of steel reinforcement that must be replaced is between the two seats shown below. If the sector area is 28.25 square feet and the radius is 12 feet, what is the length of steel that must be replaced? Describe the steps you used to find your answer.

• If the amusement park owners wanted to decorate each sector of this Ferris wheel with a different color of fabric, how much of each color fabric would they need to purchase? The area to be covered is described by an arc length of 5.9 feet. The circle has a radius of 15 feet. Describe the steps you used to find your answer.



Assessments (Pre, Formative, Summative, Other) *Denote required common assessments with an **

Pre-Assessments

Reflection is a process for looking back and integrating new knowledge. Reflections need to occur throughout the building blocks of constructivism and include teacher-led student-driven and teacher reflections. Some methods of reflection include but are not limited to:

- **Closing Circle –** A quick way to circle around a classroom and ask each student to share one thing they now know about a topic or a connection that they made that will help them to remember or how this new knowledge can be applied in real life.

- **Entrance/Exit Cards –** An easy 5 minute activity to check student knowledge before, during and after a lesson or complete unit of study. Students respond to 3 questions posed by the teacher. Teachers can quickly read the responses and plan necessary instruction.
- **Learning Logs** – Short, ungraded and unedited, reflective writing in learning logs is a venue to promote genuine consideration of learning activities.
- **Rubrics** – Students take time to self-evaluate and peer-evaluate using the rubric that was given or created at the beginning of the learning process. By doing this, students will understand what areas they were very strong in and what areas to improve for next time.

Formative Assessments

- Observation
- Homework
- Class Participation
- DO-NOWs
- Notebook

NJ Student Learning Standards Educational Resources / Tasks
http://www.state.nj.us/education/cccs/frameworks/math/g.pdf

Summative Assessments

- Chapter/Unit Test
- Quizzes
- Presentations
- Unit Projects
- Quarterly Benchmark* Testing including Mid-Term* and Final* Exams
  - Marking Period 1 Test (UNIT 1: Congruence, Proof and Constructions & UNIT 2: Similarity, Proof and Trigonometry)
  - Mid-term (UNIT 1: Congruence, Proof and Constructions, UNIT 2: Similarity, Proof and Trigonometry, and UNIT 3: Extending to Three Dimensions)
  - Marking Period 3 Test (UNIT 4: Connecting Algebra and Geometry through Coordinates & UNIT 5: Applications of Probability)
  - Final Exam (UNIT 4: Connecting Algebra and Geometry through Coordinates, UNIT 5: Applications of Probability, and UNIT 6: Circles With and Without Coordinates)

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Follow all IEP modifications/504 plan

Teaching and Learning Activities	
	Suggested Instructional Strategies: • Given any two circles in a plane, show that they are related by dilation. Guide students to discover the center and scale factor of this dilation and make a conjecture about all dilations of circles. • Starting with the special case of an angle inscribed in a semicircle, use the fact that the angle sum of a triangle is 180° to show that this angle is a right angle. Using dynamic geometry, students can grab a point on a circle and move it to see that the measure of the inscribed angle

	passing through the endpoints of a diameter is always 90°. Then extend the result to any inscribed angles. For inscribed angles, proofs can be based on the fact that the measure of an exterior angle of a triangle equals the sum of the measures of the nonadjacent angles. Consider cases of acute or obtuse inscribed angles. ● Use formal geometric constructions to construct perpendicular bisectors of the sides and angle bisectors of a given triangle. Their intersections are the centers of the circumscribed and inscribed circles, respectively. ● Constructing tangents to a circle from a point outside the circle is a useful application of the result that an angle inscribed in a semicircle is a right angle. ● When applying the Pythagorean Theorem to find the length of a radius, extend problems to find the missing length of a tangent. Incorporate some use of quadratics, squaring binomials and factoring. ● Incorporate Special Right Triangles, trigonometry when finding the measure of chords. It is a good opportunity to spiral material into lesson. ● When finding the lengths of segments in circles, incorporate quadratics. (factoring and solving) ● Begin by calculating lengths of arcs that are simple fractional parts of a circle (e.g. 1/6), and do this for circles of various radii so that students discover a proportionality relationship. ● Provide plenty of practice in assigning radian measure to angles that are simple fractional parts of a straight angle. ● Stress the definition of radian by considering a central angle whose intercepted arc has its length equal to the radius, making the constant of proportionality 1. Students who are having difficulty understanding radians may benefit from constructing cardboard sectors whose angles are one radian. Use a ruler and string to approximate such an angle. ● Compute areas of sectors by first considering them as fractional parts of a circle. Then, using proportionality, derive a formula for their area in terms of radius and central angle. Do this for angles that are measured both in degrees and radians and note that the formula is much simpler when the angels are measured in radians. ● Derive formulas that relate degrees and radians. ● Introduce arc measures that are equal to the) measures of the intercepted central angles in degrees or radians. ● Emphasize appropriate use of terms, such as, angle, arc, radian, degree, and sector. ● Use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures. **<u>Suggested Extended Instructional Strategies for College Prep:</u>** ● Use properties of congruent triangles and perpendicular lines to prove theorems about diameters, radii, chords, and tangent lines. ● Dissect an inscribed quadrilateral into triangles, and use theorems about triangles to prove properties of these quadrilaterals and their angles. ● When applying the Pythagorean Theorem to find the length of a radius, extend problems to find the missing length of a tangent. Incorporate more advanced use of quadratics, squaring binomials and factoring. ● Extend the equation of circles to inequalities. An example of an inequality that describes the points (x, y) outside the circle that are more than three units from center (4, -2) is $(x - 4)^2 + (y + 2)^2 > 9$. The graph would be a broken circle and shaded outside the circle. An example of an inequality that describes the points (x, y) inside the circle that are less than or equal to four units from center (-3, -5) is $(x + 3)^2 + (y + 5)^2 \leq 16$. The graph would be a circle and shaded inside the circle. **<u>Suggested Learning Activities:</u>** 1. <u>Introducing Circle Similarity</u> ● Give each group of students 3 or 4 different size discs made of cardboard and a tape measure. ● Have each group fill out a chart with the measures for circumference, diameter and circum/diam.

	• Have students discuss their answers for the 3rd column. What number is a close approximation of their answers? • Ask students what would happen if a circle's diameter was doubled? Why? Have students prove answer mathematically. • Ask students would the same happen if the diameter was multiplied by another number. Why? 2. <u>Introducing Central angles</u> • Distribute analog clock faces worksheet, scissors and brads. • Have each group put together their clock. • Tell the students that the brad is the vertex of the angle. • Remind students that there are 360˚ in a circle. • Have them determine the degrees in 1 minute. • Give the students a time to show on their clocks and determine how many degrees are in the angle. Repeat with 2 other times to make sure they understand concept. • Ask students if the minute hand were extended, would the distance between the two hands at the top equal the angle in the center? Explain. 3. <u>Introducing Area of a Sector</u> • Distribute analog clock faces worksheet, scissors and brads. • Have each group put together their clock. • Tell the students that the brad is the vertex of the angle. • Remind students that there are 360˚ in a circle. • Give students a time to show on their clocks & ask them what **part** of the **whole area** of the clock face is shown between the 2 hands. • Give groups 5-7 minutes to discuss & try out ideas for finding solutions. • Have someone from each group write their method of arriving at the answer. Compare methods and use discussion as basis for writing the formula using degrees. • Give students another time to show on the clock to do the same. 4. <u>Introducing Arc Length</u> • Distribute blank 20 dot circle templates from: http://nrich.maths.org/6676 and two different colored highlighters. • Give students the diameter length for each of the four circles (different measure). • Have students calculate the circumference of each circle. • Have students draw a different central angle on each of the four circles. • Have students highlight the intercepted arc. • Ask students to find the length of this **part** of the **whole circumference.** • Give the students 5-7 minutes to discuss and try out ideas to calculate the answer. Have someone from each group write their method of arriving at the answer. • Compare methods and use discussion as basis for writing the formula using degrees. • Give students 10 minutes to find the lengths for the other 3 circles on the worksheet. 5. <u>Introducing Inscribed Angles</u> • Distribute blank 12 dot circle templates from: http://nrich.maths.org/6676 , rulers and protractors. • Have the students label the dots like a clock. • Have the student make a vertex at the 9 on all of the clocks. Then draw 2 rays-one connecting to the 2 and one connecting to the 4. • Remind students that a circle has 360˚, and then ask them to find the measure of the intercepted arc. • Ask students whether the intercepted angle will have the same measure as its arc. Then have them write down a guess before measuring it with the protractor. • Have students make a different intercepted angle on another circle, find the arc measure mathematically then find the angle measure using the protractor. • Continue with the next circle. When students complete their work, have them make a t-table with angle measures and arc measures and look for the relationship between the 2 columns. • Write down each group's conclusion about the relationship. Write the rule from their ideas. • Use the graphing calculator to show that a triangle inscribed in a semicircle is a right triangle; to show that the product of the parts of one chord equal the product of the parts of the other chord;

to graph and identify circles as tangent, intersecting, or concentric; and to graph and recognize tangents as internal or external.

- Use patty paper to demonstrate the properties of circles.
- Students use post-it notes to identify intercepted arcs.
- Students use post-it notes to find multiple angles and arc measures in circle drawings.
- TI-Nspire Activity: Circles – Angles and Arcs: http://education.ti.com/calculators/timathnspired/US/Activities/Detail?sa=5024&t=5047&id=13172
- Mathematical investigations using a circular geoboard.
- Design window art using a compass and straightedge.
- Students measure circumference of hula hoop and tire then calculate distance traveled in groups.
- Students model a merry go round's motion and draw conclusions about rpms vs. mph.
- A Slice of Pi: This project studies how π has been computed throughout history, including current connections between π and geometry. http://www.geom.uiuc.edu/~huberty/math5337/groupe/welcome.html
- Exploring Geometry with The Geometer's Sketchpad: This collection of Sketchpad activities is aligned to the NJ Student Learning Standards, and covers virtually every concept studied in high school geometry. The main focus for this unit is in Chapter 6 of the module:
    - Introducing Circles pg 201
    - Chords in a Circle pg 204
    - Tangents to a Circle pg 207
    - Tangent Segments pg 209
    - Arcs and Angles pg 211
    - The Circumference/Diameter Ratio pg 214
    - The Cycloid pg 217
- Show the video "The Story of Pi" by Project Mathematics: http://www.projectmathematics.com/storypi.htm
- GeoGebra utilities: A free mathematics software for learning and teaching geometry. The following are some examples of useful interactive resources found on the website: http://www.geogebra.org/cms/
    - A Piece of Pastry! Circle Geometry Problem by Micky Bullock
    - Tangents and Radii by Laura Rees-Hughes
    - Congruent Chords and their Central Angles by Tara Sharkey
    - Apollonius Problem by Patrick Honner
    - Math 9: Chord Properties by Loucks
- SMART Exchange: Smarttech offers a variety of activities and lessons which have already been created and are ready for use. Merely enter "Circles" to search for valuable resources. Some notable activities pertaining to circles are Circles (Gallery Collection), Segment lengths and circle theorems (SMART Notebook Math Tools lesson), Angles and Arcs from Circle Theorems (SMART Notebook Math Tools lesson), etc. http://exchange.smarttech.com/search.html?subject=Mathematics
- Explorer Learning Activities such as Circles, Circles and Chords, and Area and Circumference. http://www.explorelearning.com/index.cfm?CourseID=127&method=cResource.dspChildrenForCourse
- Interdisciplinary Connections with realistic implications: 1) research the fountains of Versailles and determine the cost of restorations on the fountains; 2) a piece of circular plate was recently dug up on an island in the Mediterranean. The discoverer of the plate wishes to calculate the diameter of the original plate. Describe how he could do this; or 3) create a new clock design of the Floral Clock in Niagara Falls, Canada, given the circumference. Students need to determine the lengths of the hands of the clock, the area of each of the 12 sectors in which the letters to spell out Niagara Parks, etc.

Common Misconceptions

- Students sometimes confuse inscribed angles and central angles.
- Students may think they can tell by inspection whether a line intersects a circle in exactly one point. It may be beneficial to formally define a tangent line as the line perpendicular to a radius at the point where the radius intersects the circle.

	• Sectors and segments are often used interchangeably in everyday conversation. Care should be taken to distinguish these two geometric concepts. • The formulas for converting radians to degrees and vice versa are easily confused. Knowing that the degree measure of a given angle is always a number larger than the radian measure can help students use the correct unit.
Differentiation Strategies	Differentiation strategies may include, but are not limited to, learning centers and cooperative learning activities in either heterogeneous or homogeneous groups, depending on the learning objectives and the number of students that need further support and scaffolding, versus those that need more challenge and enrichment. Activities and resources are provided above and below. Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	The students enrolled in this level will be assigned questions of greater complexity.

Resources

Resources/Instructional Materials

- Textbook: Geometry by Holt McDougal(2012) - Teachers' Edition & accompanying resources, e.g. Transparencies, practice worksheets, assessments, writing assignments
- NJ Student Learning Standards
  http://www.state.nj.us/education/cccs/2016/math/standards.pdf
- Geometers Sketchpad
- Exploring Geometry with The Geometer's Sketchpad Activity Module
  http://www.keycurriculum.com/resources/sketchpad-resources/activity-modules/high-school-activity-modules-for-the-geometers-sketchpad#geometry
- Geogebra
  http://www.geogebra.org/cms/
- Explorer Learning
  http://www.explorelearning.com
- HSPA Mathematics Workbooks
- Circular Geoboard for the overhead projector
- Compass (for drawing circles)
- Compass (for directions)
- Straight edge
- Ruler
- Protractor

Interactive applets:

http://www.mathwarehouse.com/geometry/circle/interactive-central-angle-of-circle.phphttp://www.mathwarehouse.com/geometry/circle/inscribed-angle.php

http://mathsclass.net/geogebra/fs/parts-of-a-circle

http://www.mrperezonlinemathtutor.com/CARFILES/Tangent_and_a%20Secant_Intersecting_at_an_Exterior_Point_by_Mr_Perez.html http://www.mathwarehouse.com/geometry/circle/index.php (inscribed angle & its arc, area & circumference, central angle, products of segments theorem, intersecting chords, arc of a circle, angles made by tangent and chord

Templates for activities:

http://www.teacherled.com/resources/clockspin/clockspinload.html (analog clock for introducing central angles)

http://www.teachervision.fen.com/tv/printables/scottforesman/Math_3_TTT_15.pdf

http://nrich.maths.org/6676 (a variety of useful templates)

Wayne School District
Curriculum

Content Area/ Grade Level/ Course:	**Mathematics** **9/10** **Geometry**
Unit Plan Title:	**Unit 5: Geometric Measurement and Dimension**
Time Frame	Approximately 3-4 weeks.

Anchor Standards/Domain

G-GMD Geometric Measurement and Dimension

G-MD Modeling with Geometry

Unit Summary

The unit will begin by looking at formulas for two dimensional shapes. Visualization of the relationships between two-dimensional and three-dimensional objects will be achieved by rotating the two dimensional shape to form three dimensional solids. The formulas for the three dimensional solids will be explained and then used to solve problems of various types. The unit will explore and define many three-dimensional solids, discover volume formulas for prisms, pyramids, cylinders, cones and spheres and surface area of a sphere.

Standard Number(s)

G-GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G-GMD.A.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

G-GMD.A.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

G-MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

G-MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

G-MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

G.GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

8.1.12.CS.4 - Develop guidelines that convey systematic troubleshooting strategies that others can use to identify and fix errors.

8.1.12.DA.1 - Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.

8.1.12.DA.5 - Create data visualizations from large data sets to summarize, communicate, and support different interpretations of real-world phenomena.

8.1.12.DA.6 - Create and refine computational models to better represent the relationships among different elements of data collected from a phenomenon or process.

9.4.12.CI.1 - Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g., 1.1.12prof.CR3a).

9.4.12.CT.2 - Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a)

9.4.12.TL.2 - Generate data using formula-based calculations in a spreadsheet and draw conclusions about the data.

9.4.12.IML.3 - Analyze data using tools and models to make valid and reliable claims, or to determine optimal design solutions (e.g., S-ID.B.6a., 8.1.12.DA.5, 7.1.IH.IPRET.8)

NJSLSA.SL1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

HS-ETS1-2. Design a solution to a complex real-world problem by breaking it down into smaller, more manageable problems that can be solved through engineering.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

RST.9-10.3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

RST.9-10.4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

RST.9-10.5. Analyze the relationships among concepts in a text, including relationships among key terms (e.g.,force,, friction, reaction force, energy).

RST.9-10.7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

WHST.9-10.4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

WHST.9-10.5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.

WHST.9-10.6. Use technology , including the Internet, to produce, share, and update writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.

Essential Questions:

Students will focus on the following essential questions:

- What are the lateral area, surface area, and volume of the following figures: prisms, cylinders, pyramids, cones, and spheres?
- How do geometric models describe spatial relationships?
- How are geometric shapes and objects classified?
- What is the relationship of the different measures in two and three dimensional objects?
- How does a change in one dimension of an object affect the other dimensions?
- How can isometric drawings and cross sections be used to represent 3-dimensional figures?
- What is the difference between a pyramid and a prism? Between a face and a base?
- What are the methods for calculating the surface areas of prisms, cylinders, pyramids, cones, and spheres?
- What methods can be used to find the volumes of prisms, pyramids, cylinders, cones and spheres?
- What methods can be used to find the volumes of prisms, pyramids, cylinders, cones and spheres?
- How can the distance and midpoint formulas be applied in 3-dimensional space?
- How are two-dimensional relationships connected to the properties of three-dimensional figures?
- How are two-dimensional measurement concepts used to calculate measures of three-dimensional figures?
- What makes a three-dimensional figure a polyhedron?
- Where do figures differ in regard to calculating volume?
- How does volume relate to density?
- How do we calculate volume using only displacement?
- How do you derive the distance formula?
- How do you find the perimeter and area of a polygon given only its coordinates?

Enduring Understandings

Students will understand…

- The surface area of a three-dimensional object is the sum of the areas of all its faces.
- The volume of a three-dimensional object is the number of unit cubes that would fill the object.
- Geometry and spatial sense offer ways to interpret and reflect on our physical environment.
- Analyzing geometric relationships develops reasoning and justification skills.
- Reasoning and/or proof can be used to verify or refute conjectures or theorems in geometry.
- Solid geometry studies the surface of a three-dimensional figure and the space it encloses.
- Different characteristics create different categories of solids, and that these characteristics create different formulas for three-dimensional figures.
- There is a direct connection between the base area of many solids and its volume.
- Density is a measure that allows us to relate mass and volume of a figure.

In this unit plan, the following 21st Century themes and skills are addressed.

Check all that apply. **21st Century Themes**		*Indicate whether these skills are **E**-Encouraged, **T**-Taught, or **A**-Assessed in this unit by marking **E, T, A** on the line before the appropriate skill.* **21st Century Skills**	
X	**Global Awareness**	**T,E,A**	**Creativity and Innovation**
	Environmental Literacy	**T,E,A**	**Critical Thinking and Problem Solving**
	Health Literacy	**T,E,A**	**Communication**
X	**Civic Literacy**	**T,E,A**	**Collaboration**
X	**Financial, Economic, Business, and Entrepreneurial Literacy**		

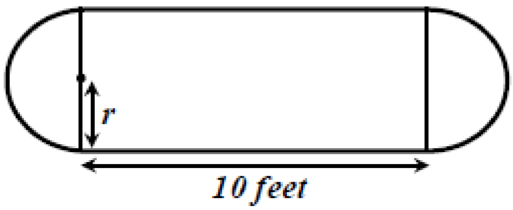
Student Learning Targets/Objectives *Exemplars attached

Students will:

1) Classify three-dimensional figures.
2) Identify parts of three-dimensional figures.
3) Calculate volume of irregular figures based upon displacement.
4) Use displacement to determine what type of material a solid is made of.
5) Using Cavalieri's principle, develop formulas for finding the surface area and volume of figures with constant width and non-constant width, along with spheres.

Explanation: Cavalieri's principle is if two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

Example:
People who live in isolated or rural areas have their own tanks of natural gas to run appliances like stoves, washers, and water heaters. These tanks are made in the shape of a cylinder with hemispheres on the ends.



The Insane Propane Tank Company makes tanks with this shape in different sizes. The cylinder part of every tank is exactly 10-feet long, but the radius of the hemispheres, rr, will be different depending on the size of the tank.
The company wants to double the capacity of their standard tank, which is 6 feet in diameter. What should the radius of the new tank be? Explain your thinking and show your calculations.

6) Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Explanation: Missing measures can include but are not limited to slant height, altitude, height, diagonal of a prism, edge length, and radius.

Example:
• Janine is planning on creating a water-based centerpiece for each of the 30 tables at her wedding reception. She has already purchased a cylindrical vase for each table. The radius of the vases is 6 cm, and the height is 28 cm. She intends to fill them half way with water and then add a variety of colored marbles until the waterline is approximately three-quarters of the way up the cylinder. She can buy bags of 100 marbles in 2 different sizes, with radii of 9 mm or 12 mm. A bag of 9 mm marbles costs $3, and a bag of 12 mm marbles costs $4.

a. If Janine only bought 9 mm marbles how much would she spend on marbles for the whole reception? What if Janine only bought 12 mm marbles? (Note: 1 cm3=1 ml)
b. Janine wants to spend at most d dollars on marbles. Write a system of equalities and/or inequalities that she can use to determine how many marbles of each type she can buy.
c. Based on your answer to part b., how many bags of each size marble should Janine buy if she has $180 and wants to buy as many small marbles as possible?

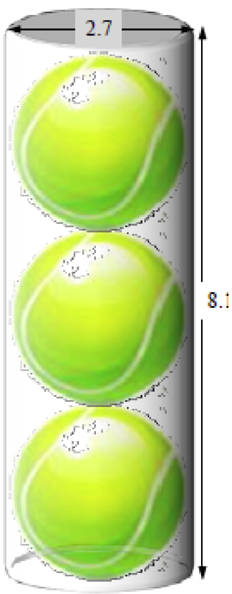
7) Classify solids as polyhedrons, and then classify polyhedrons based upon their bases and lateral surfaces.
8) Utilize Euler's formula to find the vertices, faces, and edges of a polyhedron.
9) Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Examples*:*

• Identify the shape of the vertical, horizontal, and other cross sections of a cylinder.

• The official diameter of a tennis ball, as defined by the International Tennis Federation, is at least 2.575 inches and at most 2.700 inches. Tennis balls are sold in cylindrical containers that contain three balls each. To model the container and the balls in it, we will assume that the balls are 2.7 inches in diameter and that the container is a cylinder the interior of which measures 2.7 inches in
diameter and $3 \times 2.7 = 8.1$ inches high.

a. Lying on its side, the container passes through an X-ray scanner in an airport. If the material of the container is opaque to X-rays, what outline will appear? With what dimensions?
b. If the material of the container is partially opaque to X-rays and the material of the balls is completely opaque to X-rays, what will the outline look like (still assuming the can is lying on its side)?
c. The *central axis* of the container is a line that passes through the centers of the top and bottom. If one cuts the container and the balls by a plane passing through the central axis, what does the intersection of the plane with the container and balls look like? (The intersection is also called a *cross section*. Imagine putting the cut surface on an inkpad and then stamping a piece of paper. The stamped image is a picture in the intersection.)
d. If the can is cut by a plane parallel to the central axis, but at a distance of 1 inch from the axis, what will the intersection of this plane with the container and balls look like?
e. If the can is cut by a plane parallel to one end of the can—a horizontal plane—what are the possible appearances of the intersections?
f. A cross section by a horizontal plane at a height of $1.35+w$ inches from the bottom is made, with $0<w<1.35$ (so the bottom ball is cut). What is the area of the portion of the cross section inside the container but outside the tennis ball?
g. Suppose the can is cut by a plane parallel to the central axis but at a distance of w inches from the axis ($0<w<1.35$). What fractional part of the cross section of the container is inside of a tennis ball?



10) Use geometric simulation software to model figures and create cross sectional views.
11) Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Example*:*

• Suppose that even in perfect visibility conditions, the lamp at the top of a lighthouse is visible from a boat on the sea at a distance of up to 32 km.

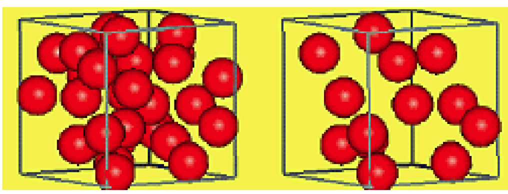
a. If the “distance” in question is the straight-line distance from the lamp itself to the boat, what is the height above sea level of the lamp on top of the lighthouse?

b. What are two other interpretations of the distance being investigated in this problem? Describe how to solve the alternate versions

12) Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Example:

• Take a look at the two boxes below. Each box has the same volume. If each ball has the same mass, which box would weigh more? Why?



Block I
Mass = 79.4 grams
Volume = 29.8 cubic cm

Block II
Mass = 25.4 grams
Volume = 29.8 cubic cm

13) Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Example:

• You have been hired by the owner of a local ice cream parlor to assist in his company’s new venture. The company will soon sell its ice cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use a real ice cream cone or the dimensions of a real ice cream cone to complete the following tasks.

a. Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly.

b. Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of length, rr, and a slant height, s.

c. Using measurements of the radius of the base and slant height of your cone and your equation from the previous step, find the surface area of your cone.

d. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.

Assessments (Pre, Formative, Summative, Other) ***Denote required common assessments with an ****

Pre-Assessments

Reflection is a process for looking back and integrating new knowledge. Reflections need to occur throughout the building blocks of constructivism and include teacher-led student-driven and teacher reflections. Some methods of reflection include but are not limited to:

- **Closing Circle –** A quick way to circle around a classroom and ask each student to share one thing they now know about a topic or a connection that they made that will help them to remember or how this new knowledge can be applied in real life.
- **Entrance/Exit Cards –** An easy 5 minute activity to check student knowledge before, during and after a lesson or complete unit of study. Students respond to 3 questions posed by the teacher. Teachers can quickly read the responses and plan necessary instruction.
- **Learning Logs** – Short, ungraded and unedited, reflective writing in learning logs is a venue to promote genuine consideration of learning activities.
- **Rubrics** – Students take time to self-evaluate and peer-evaluate using the rubric that was given or created at the beginning of the learning process. By doing this, students will understand what areas they were very strong in and what areas to improve for next time.

Formative Assessments

- Observation
- Homework
- Class Participation
- DO-NOWs
- Notebook

NJ Student Learning Standards Educational Resources / Tasks
http://www.state.nj.us/education/cccs/frameworks/math/g.pdf

Summative Assessments

- Chapter/Unit Test
- Quizzes
- Presentations
- Unit Projects
- Quarterly Benchmark* Testing including Mid-Term* and Final Exams*
  - Marking Period 1 Test (UNIT 1: Congruence, Proof and Constructions & UNIT 2: Similarity, Proof and Trigonometry)
  - Mid-term (UNIT 1: Congruence, Proof and Constructions, UNIT 2: Similarity, Proof and Trigonometry, and UNIT 3: Extending to Three Dimensions)
  - Marking Period 3 Test (UNIT 4: Connecting Algebra and Geometry through Coordinates & UNIT 5: Applications of Probability)
  - Final Exam (UNIT 4: Connecting Algebra and Geometry through Coordinates, UNIT 5: Applications of Probability, and UNIT 6: Circles With and Without Coordinates)

Modifications (ELLs, Special Education, Gifted and Talented)

- Teacher tutoring
- Peer tutoring
- Cooperative learning groups
- Modified assignments
- Differentiated instruction
- Follow all IEP modifications/504 plan

Teaching and Learning Activities

Activities	**<u>Suggested Instructional Strategies:</u>** ● Revisit formulas C=πd and C=2πr . Observe that the circumference is a little more than three times the diameter of the circle. Briefly discuss the history of this number and attempts to compute its value. ● Use alternative ways to derive the formula for the area of the circle A=πr^2. For example, cut a cardboard circular disk into 6 congruent sectors and rearrange the pieces to form a shape that looks like a parallelogram with two scalloped edges. Repeat the process with 12 sectors and note how the edges of the parallelogram look “straighter.” Discuss what would happen in the case as the number of sectors becomes infinitely large. Then calculate the area of a parallelogram with base ½ C and altitude *r* to derive the formula A=πr^2. ● Wind a piece of string or rope to form a circular disk and cut it along a radial line. Stack the pieces to form a triangular shape with base C and altitude r. Again discuss what would happen if the string became thinner and thinner so that the number of pieces in the stack became infinitely large. Then calculate the area of the triangle to derive the formula A=πr^2. ● Introduce Cavalieri’s principle using a concrete model, such as a deck of cards. Use Cavalieri’s principle with cross sections of cylinders, pyramids, and cones to justify their volume formulas. ● For pyramids and cones, the factor 1/3 will need some explanation. An informal demonstration can be done using a volume relationship set of plastic shapes that permit one to pour liquid or sand from one shape into another. Another way to do this for pyramids is with Geoblocks. The set includes three pyramids with equal bases and altitudes that will stack to form a cube. An algebraic approach involves the formula for the sum of squares ($1^2 + 2^2 + \ldots + n^2$). ● After the coefficient 1/3 has been justified for the formula of the volume of the pyramid (A = 1/3Bh), one can argue that it must also apply to the formula of the volume of the cone by considering a cone to be a pyramid that has a base with infinitely many sides. ● The formulas for volumes of cylinders, pyramids, cones and spheres can be applied to a wide variety of problems such as finding the capacity of a pipeline; comparing the amount of food in cans of various shapes; comparing capacities of cylindrical, conical and spherical storage tanks; using pyramids and cones in architecture; etc. Use a combination of concrete models and formal reasoning to develop conceptual understanding of the volume formulas. ● Review vocabulary for names of solids (e.g., right prism, cylinder, cone, sphere, etc.). ● Slice various solids to illustrate their cross sections. For example, cross sections of a cube can be triangles, quadrilaterals or hexagons. Rubber bands may also be stretched around a solid to show a cross section. ● Cut a half-inch slit in the end of a drinking straw, and insert a cardboard cutout shape. Rotate the straw and observe the three-dimensional solid of revolution generated by the two-dimensional cutout. ● Java applets on some web sites can also be used to illustrate cross sections or solids of revolution. ● Encourage students to create three-dimensional models to be sliced and cardboard cutouts to be rotated. Students can also make three-dimensional models out of modeling clay and slice through them with a plastic knife. ● Concrete models of solids such as cubes, pyramids, cylinders, and spheres. Include some models that can be sliced, such as those made from Styrofoam.

Suggested Extended Instructional Strategies for Enriched:

- Cavalieri's principle can also be applied to obtain the volume of a sphere, using an argument similar to that employed by Archimedes more than 2000 years ago. In this demonstration, cross sections of a sphere of radius R and a cone having radius 2R and altitude 2R are balanced against cross sections of a cylinder having radius 2R and altitude 2R.
- When an object is placed in a liquid, it causes the liquid to rise. This volume is called the object's' displacement. The volume of an irregularly shaped object can be found by measuring its displacement. Have students place a rock into a rectangular prism containing water. For example, the base of the container is 10 centimeters by 15 centimeters and when the rock is put in the prism, the water level rises 2 centimeters due to the displacement. This new "slice" of water has a volume of 300 cubic centimeters (10 ×15 × 2). Therefore the volume of the rock is 300 cubic centimeters.
- The students enrolled in this level will be assigned questions of greater complexity.

Additional Activities:

- Use Geoblocks or comparable models of solid shapes when demonstrating lessons.
- Show "Flatland: The Movie" (2007 starring Martin Sheen) or have students read "Flatland: A Romance of Many Dimensions" by E.A. Abbott. Flatland is a story about two-dimensional creatures—lines, triangles, squares, circles, and other polygons—that live on a plane. The protagonist and narrator of the story, A. Square, visits a one-dimensional land known as Lineland and is visited by a Sphere from Spaceland. After the Sphere takes A. Square on a tour of Spaceland and then returns him to Flatland, Square decides to share the "Gospel of Three Dimensions" with other Flatlanders. As a result, Square is imprisoned for life for his belief in three dimensions. It cleverly encourages readers to consider the idea of a fourth dimension by using the analogy of a two-dimensional being who is introduced to a three-dimensional world.
- Cutouts made from Honeycomb balls to identify three-dimensional objects generated by rotations of two-dimensional objects.
- Area and Volume of Geometric Shapes Activity. This hands-on activity gives the students the opportunity to practice calculating the surface area and volume of different solids. http://www.math.twsu.edu/history/activities/geometry-act.html#areavolume-act
- Making Polyhedra Activity. This activity encourages students to examine and understand the properties of polyhedral as well as Euler's formula. http://www.math.twsu.edu/history/activities/geometry-act.html#poly-act
- Exploring Geometry with The Geometer's Sketchpad: This collection of Sketchpad activities is aligned to the NJ Student Learning Standards, and covers virtually every concept studied in high school geometry:
    - The Circumference/Diameter Ratio
    - Areas of Regular Polygons
    - Constructing Templates for the Platonic Solids
    - Drawing a Box with Two-Point Perspective
    - The Burning Tent Problem
    - The Feed and Water Problem
    - Finding the Width of a River
    - Finding the Height of a Tree
    - Measuring Height with a Mirror
    - Spacing Poles in Perspective
    - Modeling a Pantograph
    - Planning a Path for a Laser
    - The Surfer and the Spotter
    - A Rectangle with Maximum Area
    - Dividing Land
- GeoGebra utilities: A free mathematics software for learning and teaching geometry. The following are some examples of useful interactive resources found on the website: http://www.geogebra.org/cms/
    - Archimedes and the Volume of a Sphere: A dynamic illustration of Archimedes derivation of the sphere volume formula.
    - Comparison of Surface Area and Volume of Cubes
    - Right Circular Cone
    - Volume of Sphere Compared to Volume of Hemisphere

- SMART Exchange: Smarttech offers a variety of activities and lessons which have already been created and are ready for use. Merely enter “3-D, Volume, Polyhedra, etc.” to search for valuable resources. Some notable lessons/activities pertaining to this unit are Volume (SMART Notebook lesson), Circumference, Volume, Surface Area (SMART Notebook lesson), Geometry Surface Area and Volume of Spheres (SMART Notebook lesson), Geometry Volume of Pyramids and Cones (SMART Notebook lesson), Volume:Q1 (SMART interactive Response question), etc. http://exchange.smarttech.com/search.html?subject=Mathematics
- Explorer Learning Activities such as 3D and Orthographic Views - Activity A & B, Prisms and Cylinders - Activity A, and Surface and Lateral Area of Pyramids and Cones, etc. http://www.explorelearning.com/index.cfm?CourseID=127&method=cResource.dspChildrenForCourse
- “Hands-On” Volume and Surface Area Activities:
  - Cylinder: For volume and total surface area, I used a Pringles can, emptied and cut vertically and 2/3 of the way around the circumference of the base (so it stays attached). I taped the plastic top to the box to be the top or bottom (depending on your perspective). It was something the kids could unroll and roll up again, and that I could refer to repeatedly throughout the unit. The label of any soup can is a clear example of lateral surface area if you need it.
  - Rectangular box: I used a box that had some extra tabs that made it easy to view flat or in 3D, but any rectangular box that you can cut so that the net is easy to see is good. I told students that when I saw a problem with a tall, skinny box, I thought of a cereal box. While I didn't use one, it would be another good example to cut and show. I would also suggest cutting a box that is a common sight in the room (tissue box, printer paper box, etc) which is easiest for you to get and easy for students to use as a reference.
  - Triangular prism: Toblerone, the oddly-shaped Swiss chocolate bar, is one of the few triangular-shaped retail boxes that is both widely available and immediately recognizable. I had two boxes, one taped together (after I removed the chocolate to avoid any distractions) and another cut to form a easy-to-sketch net. A large 3-ring binder is another potential example hiding in your classroom.
- Cabri is a simple and comprehensive software to understand 3D geometry in the classroom. The 3-D gallery offers students the opportunity to interact and manipulate topics on areas of Measurements & Equations, Polyhedrons, Space Geometry, and Optics.
  http://www.cabri.com/
- Nets, Surface Area & Volume: Student Activity Lesson Plan:
  http://www.learnalberta.ca/content/mejhm/html/object_interactives/surfaceArea/use_it.html (teacher demo)
  http://www.learnalberta.ca/content/mejhm/html/object_interactives/surfaceArea/explore_it.html (student use)

Common Misconceptions

- An informal survey of students from elementary school through college showed the number pi to be the mathematical idea about which more students were curious than any other. There are at least three facets to this curiosity: the symbol π itself, the number 3.14159…, and the formula for the area of a circle. All of these facets can be addressed here, at least briefly.
- Many students want to think of infinity as a number. Avoid this by talking about a quantity that becomes larger and larger with no upper bound.
- The inclusion of the coefficient 13 in the formulas for the volume of a pyramid or cone and 43in the formula for the volume of a sphere remains a mystery for many students. In high school, students should attain a conceptual understanding of where these coefficient come from. Concrete demonstrations, such as pouring water from one shape into another should be followed by more formal reasoning.
- Generating solids of revolution involves motion and is difficult to visualize by merely looking at drawings.
- Some cross sections are more difficult to visualize than others. For example, it is often easier to visualize a rectangular cross section of a cube than a hexagonal cross section.

	G-GPE.B.7 (1) (Polygon Area Calculator) (Coordinate Geometry) http://www.mathopenref.com/coordpolygonareacalc.html (2) Area and Perimeter of a square (Coordinate Geometry) **http://www.mathopenref.com/coordsquareareaperim.html**
Differentiation Strategies	Differentiation strategies may include, but are not limited to, learning centers and cooperative learning activities in either heterogeneous or homogeneous groups, depending on the learning objectives and the number of students that need further support and scaffolding, versus those that need more challenge and enrichment. Activities and resources are provided above and below. Differentiation Strategies for Special Education Students Differentiation Strategies for Gifted and Talented Students Differentiation Strategies for ELL Students Differentiation Strategies for At Risk Students
Honors	The students enrolled in this level will be assigned questions of greater complexity.